Dessler: 1. I do not understand the shock dissipation mechanism sufficiently well to object to your suggestion that the shock heating process operates farther out in the tail. 2. I do not consider the comments made by Smith have raised any difficulties. Perhaps only a few shocks are necessary to form the ring current.

Gold, T.: Only the *a-c* part of the energy content of the solar stream is used in your case for generating the effect. This must be less than the *d-c* energy content, and could be a great deal less. In that case there would be a shortage of energy.

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INTERNATIONAL CONFERENCE ON COSMIC RAYS AND THE EARTH STORM Part I

I-3-5. Variation of Upper Atmosphere Densities with Solar Activity

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When an electric field ($\boldsymbol{E}$) exists orthogonal to a magnetic field ($\boldsymbol{F}$) in ionised gas the current $j=\sigma_4 E$ where $\sigma_4{}^2=\sigma_1{}^2+\sigma_2{}^2$ where σ_1, σ_2 are the Pedersen, Hall conductivities. The joule heating $Q=j^2/\sigma_3$, where $\sigma_3=\sigma_1+\sigma_2{}^2/\sigma_1$ is the Cowling conductivity, is usefully expressed as $Q=\sigma_1 E^2$.

For study of geomagnetic disturbance, geomagnetic field lines in and above the ionosphere are regarded as equipotentials of the electric field. Hence j has a broad peak from about 110 km to 150 km altitude and Q has a broad peak from about 130 km to 170 km.

The decay time of air motion across the geomagnetic field is $\tau_v=\rho/\sigma_1 F^2$ where ρ is the density. Thus above about 130 km convection across the geomagnetic field is largely inhibited. The joule heating of the atmosphere due to disturbance currents peaks where σ_1 peaks, *i.e.* about 150 km. This heat source makes the air expand upwards along the magnetic field. Thus the scale heights above 130 km are greater over the auroral zones than at other latitudes. From $\nabla p=\boldsymbol{j}\times\boldsymbol{F}$ it follows that for moderate magnetic disturbance a pressure at 200 km altitude in the auroral zone equal to that at 120 km in other latitudes may be maintained.

Over the whole globe scale heights above 100 km increase and decrease with geomagnetic disturbance due to joule heating of disturbance currents. This affords a simple explanation of the correlation of orbital acceleration of satellites and *Kp*.

§1. Introduction

It will be shown that density scale heights in the upper atmosphere above about 100 km height increase and decrease with solar activity. This is a consequence of joule heating by those electric currents flowing in the ionosphere which cause geomagnetic disturbance. Moreover it will be shown that Lorentz forces associated with these currents are capable of supporting pressures at altitudes in excess of about 130 km over the auroral zone many times those at similar altitudes in other latitudes.

§2. Magnetic Disturbance in Upper Atmosphere

For the purpose of study of magnetic disturbance in the upper atmosphere from the lowest inosphere upwards, the lines of force of the geomagnetic field ($\boldsymbol{F}$) are considered to be equipotentials of the electric field. This is justified on the grounds that the conductivity (σ_0) parallel to $\boldsymbol{F}$ is very much greater than that across $\boldsymbol{F}$. The current caused by an electric field $\boldsymbol{E}$ orthogonal to $\boldsymbol{F}$ is

$$\boldsymbol{j}=\sigma_1\boldsymbol{E}+\sigma_2\frac{\boldsymbol{F}\times\boldsymbol{E}}{F}. \tag{1}$$

Whence, $\quad j=\sigma_4 E, \quad$ (2)

where $\quad \sigma_4=\sqrt{\sigma_1^2+\sigma_2^2}$.

The joule heating $\quad Q=\boldsymbol{j}\cdot\boldsymbol{E}, \quad$ *i.e.*

$$Q=\sigma_1 E^2. \tag{3}$$

Q may be expressed (using 2) in the well-known form j^2/σ_3 where $\sigma_3=\sigma_1+\sigma_2^2/\sigma_1$ (the Cowling conductivity).

In this paper consideration will be restricted to ionospheric regions though some of the conclusions will apply to the exosphere.

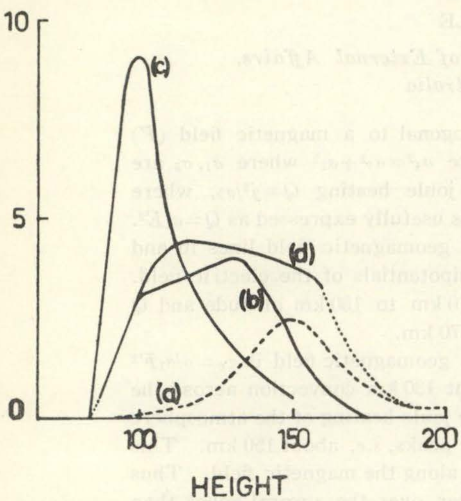

Fig. 1. Graphs of (a) $\sigma_1\times10^{15}$ e.m.u., (b) $\sigma_2\times10^{15}$ e.m.u., (c) $\sigma_3\times10^{14}$ e.m.u., taken from Fig. 2 of Chapman (1960, his *h*-model), and (d) $\sigma_4\times10^{15}$ e.m.u., calculated therefrom.

In the ionosphere the geomagnetic field lines do not separate much hence along $\boldsymbol{F}$, E is regarded as constant. Therefore in the ionosphere $Q\propto\sigma_1$ whereas $j\propto\sigma_4$. Fig. 1 shows how σ_1, σ_2, σ_3, σ_4 vary with height in Chapman's (1956)[1] model atmosphere (his Fig. 2, model *h*).

In the auroral zones E values of 10^4 e.m.u. would explain average geomagnetic disturbance (*c.f.* Akasofu, 1960)[2]. Suppose electron density $n=10^7$ (Jackson and Seddon, 1959)[3]. Then using values of σ_1/n from Chapman's *h*-model it is found that $Q=10^{-5}$, 2×10^{-5}, 10^{-5} ergs cm^{-3} sec^{-1} at 130, 145, 160 km

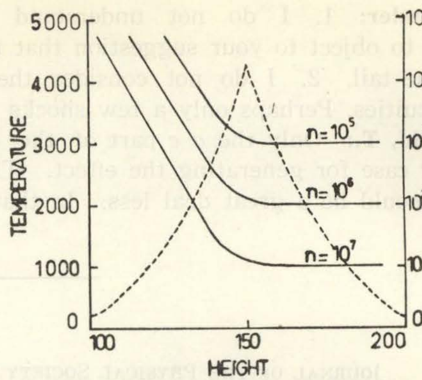

Fig. 2. Dashed curve, plot of temperature (°K) versus height (*km*) for model in text. Full lines, plot of τ_v (sec.) versus height for different values of electron density (n).

respectively. Heating of the atmosphere would proceed until the established thermal gradient induced enough conduction or convection to remove heat at the input rate.

§ 3. Spherically Symmetric Atmospheric Heating

In Chapman's *h*-model atmosphere σ_1 and therefore Q is roughly symmetrical about the 150 km level (r_0). Therefore in investigating the temperature profile that may be established in the steady state by the above process we adopt the following model for Q in an originally isothermal atmosphere.

$$Q=Q_0\exp\{(r-r_0)/H\},\quad r<r_0 \tag{4a}$$

$$Q=Q_0\exp\{(r_0-r)/H\},\quad r>r_0 \tag{4b}$$

r, r_0 are distances in cm from the centre of the earth. Spherical symmetry is assumed.

In the steady state, neglecting convection, after Nicolet (1960)[4],

$$F=4\pi r^2\cdot\lambda_c\frac{dT}{dr}, \tag{5}$$

where F is the heat flux through a concentric sphere of radius r, and λ_c ($=AT^{1/2}$) is the thermal conductivity. Thus for $r<r_0$

$$\frac{d}{dr}\left(4\pi r^2\quad AT^{1/2}\frac{dT}{dr}\right)=4\pi r^2 Q_0\exp\{(r-r_0)/H\}.$$

Providing $H/r\ll1$, which is likely to be the case,

$$\begin{aligned}T^{3/2}=T_2^{3/2}&-\frac{3}{2}\left(\frac{1}{r}-\frac{1}{r_2}\right)r_1^2T_1^{1/2}\frac{dT_1}{dr_1}\\&+\frac{3}{2}\frac{Q_0H^2}{A}[\exp\{(r-r_0)/H\}\\&-\exp\{(r_2-r_0)/H\}]\\&+\frac{3}{2}\left(\frac{1}{r}-\frac{1}{r_2}\right)r_1^2\frac{Q_0H}{A}\exp\{(r_1-r_0)/H\}\end{aligned} \tag{6}$$

Likewise T in the region $r>r_0$ is found.

Suppose $T_{1,2}$ (at 100 km height)$=225°K$, $\frac{dT_1}{dr_1}\approx 0$, $H=25$ km, and $A=180$ (*c.f.* Nicolet, 1960)[4]. Table I shows values of T_0 (at 150 km height) for various Q_0 assuming a steady state is possible. Fig. 2 (dashed curve) shows a plot of T for $Q_0=10^{-5}$ ergs cm^{-3} sec^{-1} corresponding to moderate magnetic disturbance in the auroral zones.

Table I.

Q (ergs cm^{-3} sec^{-1})	10^{-8}	10^{-7}	10^{-6}	10^{-5}	10^{-4}
T_0 (degrees K)	240	370	1,180	5,270	24,300

§ 4. Joule Heating in Limited Regions

Johnson (1960)[5] has suggested that there should be global pressure and temperature equalisation at 200 km altitude. However he did not take into account the Lorentz force of the quasi-permanent (see Fig. 3) geomagnetic disturbance currents in the ionosphere.

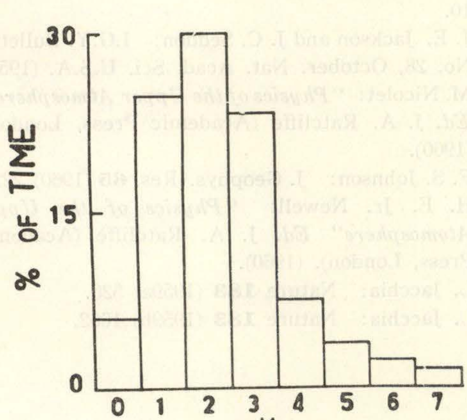

Fig. 3. Distribution of Kp as percentage of time from August to December 1958 (inclusive).

In the event of steady disturbance in the auroral zones, let the horizontal pressure gradient ∇p established by joule heating balance the Lorentz force $\boldsymbol{j} \times \boldsymbol{F}$. Then $\Delta p \cos I=\sigma_4 \cdot EF$ where I is the magnetic dip. Let us consider the height of 200 km. $\sigma_4 \approx \sigma_1$ here. Put $n=10^7$ (disturbed auroral ionosphere), $\sigma_1/n = 2 \times 10^{-21}$ e. m. u. (Chapman, 1956), $E=2\times 10^4$ e. m. u. (a moderate value), $F=0.6$ gauss, $\cos I=0.2$. σ_1/n could even be increased because the envisaged atmosphere is probably hotter than Chapman's h-model. Then ∇p is at least 2×10^{-9} dynes cm^{-3}. Thus across an auroral zone of half width 300 km (at moderate disturbance) a pressure difference at 200 km altitude of size at least 6×10^{-2} dynes cm^{-2} may be maintained. Pressures of this size are measured between 115-120 km over White Sands, New Mexico.

This point can be examined in another way. Johnson asserts that organised air motion would prohibit any major departures of p=constant surfaces from level ones. When wind of speed v (and therefore kinetic energy density $1/2\ \rho v^2$) moves across F the time rate of dissipation of energy is (from equation 3) $\sigma_1 v^2 F^2$. Therefore the exponential decay time of wind $\tau_v=\rho/\sigma_1 F^2$. Using values of σ_1/n from Chapman's (1956)[1] h-model and taking values of ρ from Fig. 5 of Newell (1960)[6], Fig. 2 (full curves) shows τ_v as a function of height for $n=10^7$, 10^6, 10^5. Since geomagnetic disturbance is quasi-permanent and has a large diurnal component let us arbitrarily select $\tau_v \gtrsim 10^3$ secs as the criterion that air motion may effectively release pressure built up by joule heating. It is seen from Fig. 2 that this may not be done above 150 km altitude even for the ordinary conditions of $n=10^5$ and that, as n increases, the height at which the criterion applies decreases.

§ 5. Discussion

It is apparent from Table I that joule heating by those electric currents in the ionosphere responsible for geomagnetic disturbance can cause great changes in scale height of the upper atmosphere. Obviously the amount of heating ranges over as many orders of magnitude as does geomagnetic disturbance. This effect will be greatest in the auroral zones at the location of the intense portions of the DS current system (Cole 1961, this Conference). The pressure built up by this process will not be released horizontally (except in equatorial regions) for the air becomes constricted by the geomagnetic field—this is indicated by the parameter τ_v. In the absence of heat loss it is of interest to represent Q values as equivalent temperature rises per second δ $(=MQ/\rho k$, where M is molecular weight and k is Boltzmann's constant). Putting $Q_0=10^{-5}$ ergs cm^{-3}

sec^{-1}, $\delta=10^{-1}$, 1, 10 degrees K sec^{-1}, at 130, 145, 160 km respectively. This scale heights would tend to double in hours, minutes, seconds respectively at these heights in a model atmosphere in which $n=10^6-10^7$ at all heights. World-wide fluctuations in scale heights (above about 120 km) due to the above process afford a simple explanation of the observed changes in orbital acceleration of satellites with changes in geomagnetic disturbance (Jacchia 1959a[7], 1959b[8]).

It follows from the theory above that the temperature versus height profile should have a bend in it near 150 km altitude. Graphs of *density* versus height (see Figs. 5 and 6, Newell, 1960)[6] have bends indicating that scale heights are an order of magnitude greater above 150 km than below. Joule heating as above may account for a substantial part of this change in scale height although part may be due to dissociation (at most a factor 2) and part to a heat source above the atmosphere. It is further suggested that the theory accounts for the remarkable difference in densities observed over Fort Churchill (auroral zone) and White Sands (low latitude) at altitudes over 100 km. (see Figs. 5 and 6, Newell, 1960)[6] even though they are not simultaneous measurements.

Assuming that the electron density in the ionosphere is a function of density rather than height, it follows that the ionosphere above about 140 km altitude should rise and fall with geomagnetic disturbance.

The geomagnetic field acts as a brake on wind movement in the ionosphere. Thus diurnal components of horizontal wind ($\tau_v\approx 10^5$ sec) may not exist above about 130 km (assuming $n\approx10^5$ electrons cm^{-3}), a quiet value. Horizontal tidal oscillations of the atmosphere must therefore be confined below this height. Above this height tidal oscillations constrained along the geomagnetic field seem feasible.

It is known that atmospheric geomagnetic disturbance currents have filamentary structure, therefore field-aligned inhomogeneities both in air density and ionisation density will project upwards from them. It may be expected that the latter will extend further upwards than the former. Depending on the nature of the electron density versus height profile before heating the field-aligned structures may have less or more ionization density than their immediate surroundings.

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3) J. E. Jackson and J. C. Seddon: I.G.Y. Bulletin, No. 28, October. Nat. Acad. Sci. U.S.A. (1959).
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6) H. E. Jr. Newell: "*Physics of the Upper Atomosphere*" *Ed.* J. A. Ratcliffe (Academic Press, London). (1960).
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8) L. Jacchia: Nature **183** (1959b) 1662.

Discussion

Martyn, D. F.: (1) There are now four conductivities, $\sigma_1\sim\sigma_4$! It is interesting to remark that σ_3 (the Cowling conductivity) which is given by $\sigma_1+\sigma_2^2/\sigma_1$, was anticipated by Clerk Maxwell in vol. II of "Electricity and Magnetism". He demonstrates there that the heating in a medium with cross conductivities is j^2/σ_3, or $\rho_1 j^2$, where j is the current density and ρ_1 the direct *resistivity*. This formula has been re-discovered, almost a century later!

Mr. Cole has assumed a field of 2×10^{-4} e.m.u. in the auroral jet, and an electron density of $10^7/cm^3$. These numbers are not consistent with the observed current densities. There may well be columns of dense ionisation with $n=10^7$, but the field across these (horizontally) will be greatly reduced by polarization, in order to keep the horizontal current uniform. I think that a consistent use of numbers for fields and conductivities would lead to a deduction of very small heating.

Cole, K. D.: Using a value of σ_4/n (conductivity per ion-electron pair) of 4×10^{-21} e.m.u., $n=10^7$ and assuming the auroral electrojet to be 1000 km wide and 50 km in height thickness, one obtains a total current of 4×10^6 amp. This is only about twice the average current in the auroral jet in an equinoctial season (Fukushima and Oguti 1953).

As regards polarization—it would require very special geometry to reduce the applied field by an order of magnitude. In any case in the above theory E is the vector sum of all electric fields, no matter how they arise.

Hines, C. O.: The rate of heating can be deduced by a method that does not depend on absolute conductivities, but only the ratio R of integrated Pederson to integrated Hall conductivities, if it is believed that the velocity of auroral motions represents $E \times B/B^2$. From this relation $|E|$ can be calculated. The magnetic measurements yield current flow. This, if multiplied by R, yields the dissipative component of the currents—the Pederson current—which in turn, when multiplied by $|E|$, yields the dissipation rate. Axford and I have done this, and derive a value which is down by only one order of magnitude from the value quoted by Cole. I do not think we can argue over one order of magnitude in such calculations.

Chamberlain, J. W.: (Regarding comments by Dr. Hines) I think auroral motions should not be used to derive the magnitude of a driving electric field, as though the velocities represented the transport velocity of atmospheric electrons. Auroral motions, both visible and on radar, are clearly not equivalent to electron motions in the atmosphere.

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I-3-6. Interchange and Rotation of the Earth Field Lines*

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I plan to discuss two theoretical points concerning the structure of the magnetosphere of the earth. One is connected with the possible motions of the thermal gas and their effects on the orbits of energetic particles, and the other is concerned with the structure of the outer atmosphere that results from the combined effects of gravitation, magnetic field and rotation.

I have discussed elsewhere (in the Journal of Geophysical Research) the possibility that the thermal gas in the magnetosphere may take part in large scale motions without any magnetic work being done or any magnetic fields being changed. This turned out to be the consequence of the insulation of the lower atomosphere which prevents any substantial vertical current through it. The patterns of motion in this class are all of the kind that might be described as an interchange of tubes of force. The rule is merely that all the gas that was at one time on a common line of force will at all future times be found on a common line of force, though not necessarily occupying the same part of space. The wide class of interchange motions that can occur have effects on the orbits of more energetic particles. Hines and Axford have discussed one class of interchange motions which they expect to occur as a consequence of the streaming solar plasma interacting with the magnetosphere. They discussed the manner in which energetic particles, in that case electrons of 5 to 50 kev, get distributed and in part precipitated in the auroral zone.

The interchange motion is associated with electric fields. It is through these fields that energetic particles have their orbits affected. One can see no good reason why interchange motions and their associated electric fields should not be very significant and indeed the

* The discussion of this paper is printed in page 204.