Discussion

Aono, Y.: How long is the distance between the stations of each group?

Lebedinsky, A.I.: Between Moscow and Sverdlovsk about 2000 km, between Moscow and Tomsk about 3000 km and between Moscow and Irkutsk about 4000 km.

JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN Vol. 17, SUPPLEMENT A-I, 1962
INTERNATIONAL CONFERENCE ON COSMIC RAYS AND THE EARTH STORM Part 1

I-5-2. Charged Particles in the Earth's Magnetic Field and the Ionospheric F2 Layer

P. ROTHWELL

Department of Electron Physics, the University of Birmingham, Birmingham 15, England

Summary

It is suggested that large fluxes of ionospheric electrons and protons spiral along magnetic lines of force of the earth's field to establish an equilibrium condition between the *F2* layers of magnetic conjugate points in opposite hemispheres such that equal fluxes of ionised particles are found at both places. Several features of the behaviour of the *F2* layer can be explained on the basis of this hypothesis, and experimental observations of *F2* layer critical frequencies provide evidence of the interdependence of the ionospheres at magnetic conjugate points.

It is further suggested that the ionosphere may be an important source of electrons trapped in the Van Allen radiation belts, if ionospheric electrons of thermal energies travelling between hemispheres are accelerated and undergo changes in pitch angle during magnetic storms, like the higher energy trapped particles.

1. The critical frequency for the reflection of radiowaves from the *E* and *F1* layers of the ionosphere generally shows a simple time dependence on the solar zenith angle. The critical frequency for the *F2* layer (f_0F2) shows much more complex variations, however, and so far no simple explanation has been found for many aspects of its behaviour, some of which are listed below:

a) The time of diurnal maximum value of (f_0F2) varies with place and season.

b) The diurnal maximum (f_0F2) max. is higher in winter than in summer.

c) The average value of (f_0F2) over a year is higher in the northern than in the southern hemisphere.

d) (f_0F2) max. varies with longitude for places at similar geographic latitudes.

e) (f_0F2) max. shows a greater dependence on geomagnetic than geographic latitude, although there is a close correlation between (f_0F2) max. and the intensity of solar electromagnetic radiation over the eleven year solar cycle.

It is suggested here that large fluxes of ionospheric protons and electrons from the *F2* layer travel along the lines of force linking two magnetically conjugate points, to establish a near equilibrium condition between the ionospheres in opposite hemispheres such that equal fluxes of ionised particles are found at both places. In this case the number of electrons and ions in the *F2* layer would depend on the ionisation processes in the upper atmosphere occurring not only at the place of observation, but also at the magnetic conjugate point in the opposite hemisphere. In other words, ionisation in the *F2* layer should depend on combined conditions at both intersections of lines of force of the earth's magnetic field with the earth's atmosphere.

2. Vestine (1960) has calculated the approximate intersections of the lines of force

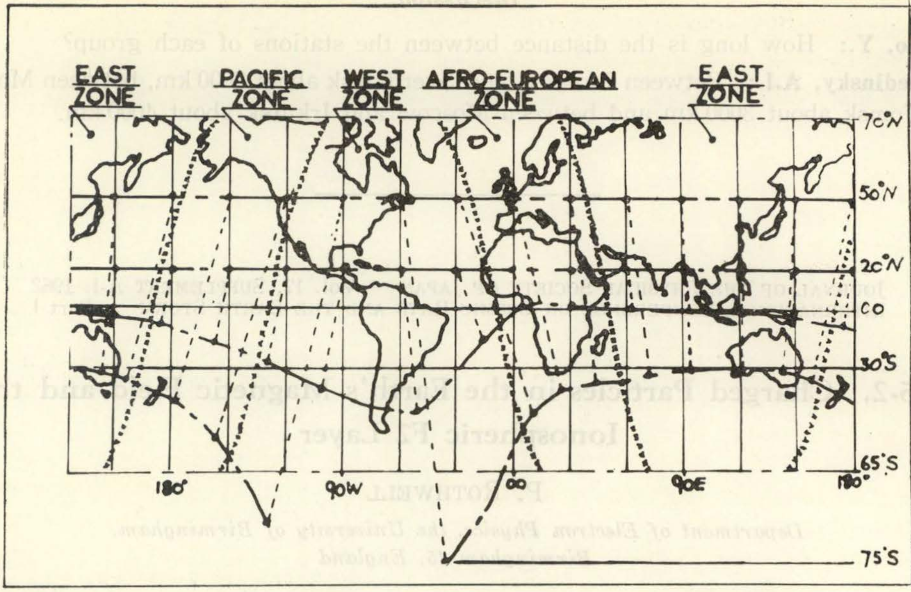

Fig. 1.

of the geomagnetic field with the earth's surface in northern and southern hemispheres, using the first 48 Gauss coefficients in the analysis of the 1955 magnetic field. Some samples of these intersections are shown in Fig. 1, which also shows the boundaries of the four 'ionospheric zones' used by ionospheric prediction services.

It is now well known that high energy particles trapped in the Van Allen radiation belts spiral up and down lines of force in the earth's magnetic field between magnetic mirror points. Low energy charged particles from the *F2* layer can also spiral along lines of force, between ionospheres at conjugate points in opposite hemispheres, since recombination times for the F_2 layer are slow (of the order of days above 400 km. (Ratcliffe, 1956). Protons or oxygen ions at 1500°K would take a few hours to travel between hemispheres along magnetic lines of force at middle latitudes (electrons would move so as to satisfy the condition of electrical neutrality at every point).

If we assume that neutral matter in space surrounding the earth is distributed according to the gravitational potential only (and the ions and electrons would together constitute a neutral plasma), then the density of ions, N_1, at r earth radii geocentric distance is given by the expression

$$N_i = N_o \exp -\left[\frac{mgR_e(1-1/r)}{kT}\right] \qquad (r<4) \qquad (1)$$

where

m =mass of the ions
R_e =radius of the earth
k =Boltzmann constant
T =ion temperature in the F_2 layer
N_o =maximum ion density in the F_2 layer.

The centrifugal force due to the earth's rotation is small compared with the gravitational force at geocentric distances less than $4R_e$ (where the line of force at geomagnetic latitude 60° cuts the equatorial plane). (At distances greater than $7R_e$-where the line of force at 68° cuts the equatorial plane-the centrifugal force is greater than the gravitational force.)

Taking $R_e=6.4\times10^8$ cm., $k=3.6\times10^{-16}$ erg. $\angle{}^\circ K$, $T=1500^\circ K$, $N_o=10^6$ions/cm.^{2}, proton mass$=1.7\times10^{-24}$, oxygen ion mass$=2.7\times10^{-23}$, equation (1) gives

$$N_i\,(\text{proton})=10^6\exp\left[-2.0\left(1-\frac{1}{r}\right)\right] \qquad (2a)$$

$$N_i\,(\text{oxygen})=10^6\exp\left[-32\left(1-\frac{1}{r}\right)\right] \qquad (2b)$$

The proton density at $r=2$, for instance (where the line of force at 45° cuts the equatorial plane) is $\sim4\times10^5$ protons/cm.^{3}, while the oxygen ion density is ~2 ions/cm.^{3}. The fluxes of ions at 1500°K which give these densities are $\sim 4\times10^{11}$ protons/cm.^{2}/sec., or 5×10^5 oxygen ions/cm.^{2}/sec.

There are $\sim 10^{13}$ ions and electrons in a cm.^{2} column of $F2$ layer, so a comparable number of charged particles should arrive from the conjugate point during the course of a day, for equilibrium to be set up between ionospheres of opposite hemispheres. This means that nett fluxes of at least 10^8 ions/cm.^{2}/sec. should pass along lines of force connecting conjugate points.

It should therefore be quite possible for protons and electrons at 1500°K to establish this equilibrium at latitudes below 60°, though it would not be possible for oxygen ions to do so.

According to Johnson (1960) the majority of positively charged particles above about 1600 km. should be protons, because hydrogen predominates over oxygen above 1600 km. and oxygen ions charge exchange easily with neutral hydrogen (at a rate greater than the gas collision cross-section) since the energy difference between their ionisation potentials is only 0.02eV.

3. It can be shown that if there are equal fluxes of ionised particles at conjugate points in opposite hemispheres, then the critical frequencies for the reflection of radio-waves from the $F2$ layer at two conjugate points should be comparable at a given universal time during the day, if the local temperatures are comparable: this can then be checked experimentally.

Critical frequencies give a measure of electron density, N_e, in the $F2$ layer (i.e. $f_0F2 \propto \sqrt{N_e}$). During hours of daylight there are roughly equal numbers of electrons and positive ions in the $F2$ layer ($N_e=N_i$), but during the night many electrons may become attached to oxygen atoms to form negative ions (Bates and Massey [1946] and Setty [1960]), and these are not detected by radio methods. Critical frequencies can be assumed to give a measure of ion density during the daytime, therefore, but not during the night.

For a given flux of ions and electrons, ϕ, the ion density N_i is obtained from the expression

$$N_i=\frac{\phi}{v}\propto\frac{\phi}{\sqrt{T}}$$

where

v is the ion velocity, and

T the ion temperature.

If

$$(f_0F2)_N, \text{ and } (f_0F2)_S$$

are values of critical frequencies at a given

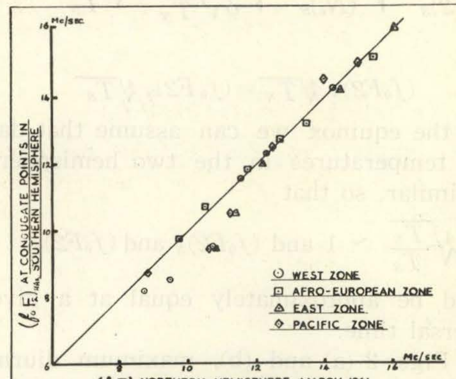

Fig. 2(a).

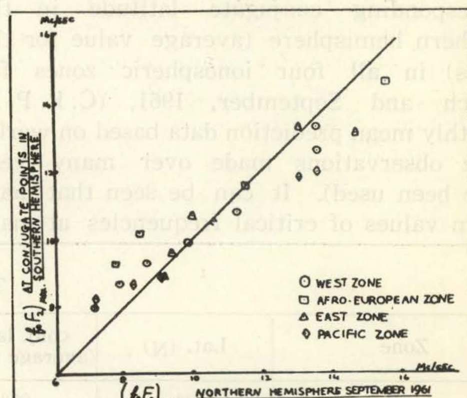

Fig. 2(b).

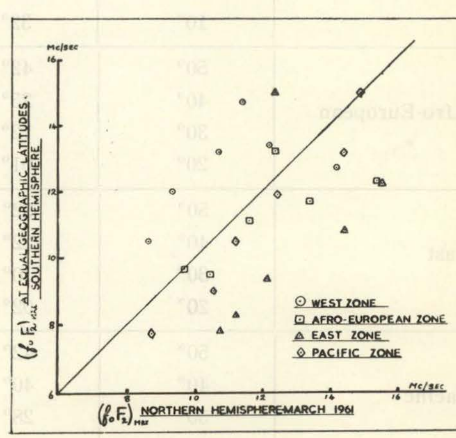

Fig. 2(c).

universal time during the day at a pair of conjugate points in the northern and southern hemispheres respectively, and T_N, T_S, are the corresponding temperatures in the $F2$ layer, then

$$\frac{(f_0F2)_N}{(f_0F2)_S}=\sqrt{\frac{(N_e)_N}{(N_e)_S}}=\sqrt{\frac{\phi\sqrt{T_S}}{\phi\sqrt{T_N}}}=\sqrt[4]{\frac{T_S}{T_N}} \quad (4)$$

i.e.

$$(f_0F2)_N\sqrt[4]{T_N}=(f_0F2)_S\sqrt[4]{T_S}$$

At the equinox we can assume that daytime temperatures in the two hemispheres are similar, so that

$$\sqrt[4]{\frac{T_N}{T_S}}\simeq 1 \text{ and } (f_0F2)_N \text{ and } (f_0F2)_S$$

should be approximately equal at a given universal time.

In Figs. 2 (a) and (b), maximum diurnal values of critical frequency, $(f_0F2)_{max}$. at 10° latitude intervals in the northern hemisphere are plotted against values of $(f_0F2)_{max}$. at the corresponding conjugate latitude in the southern hemisphere (average value for the zones) in all four ionospheric zones for March and September, 1961, (C. R. P. L. monthly mean prediction data based on worldwide observations made over many year have been used). It can be seen that maximum values of critical frequencies at magnetic conjugate latitudes in opposite hemispheres are indeed very similar. Fig. 2 (c) shows by contrast the wide variation in values of $(f_0F2)_{max}$. at equal geographic latitudes in opposite hemispheres for March, 1961.

In table I the local times of maximum $F2$ layer critical frequencies at different latitudes in the northern hemisphere $(t_N)_{max}$. are compared with the local times of maximum critical frequency at the conjugate latitudes in the southern hemisphere $(t_S)_{max}$., in each of the four different zones for March, 1961. At a given universal time, the local times at the two intersections of a line of force with the earth's atmosphere are not the same, unless the conjugate points lie near the same line of longitude. This condition is satisfied on average in the East and West zones, but in the Afro-European zone points in the northern hemisphere lie west of their conjugate in the southern hemisphere, while in the Pacific zone they lie east of them (see Fig. 1). Table I shows that the local times of $(f_0F2)_{max}$. are more nearly alike in both hemispheres in the East and West zones than in the intermediate zones. $(t_N)_{max}$. occurs before $(t_S)_{max}$. in the Afro-European zone and after $(t_S)_{max}$. in the Pacific zone. This is to be expected if the maximum diurnal values of critical frequencies occur at approximately

Table I.

Zone	Lat. (N)	conj. lat. (S) (average in zone)	t_N	t_S	(t_N-t_S)
West	40°	65°	14.00	13.30	+0.30
	30°	55°	13.00	12.30	+0.30
	20°	45°	13.00	13.00	00
	10°	32°	14.30	15.15	−0.45
Afro-European	50°	42°	12.30	14.00	−1.30
	40°	25°	11.00	13.00	−2.00
	30°	12°	11.30	13.30	−2.00
	20°	01°	14.00	14.30	−0.30
East	50°	32°	13.00	12.30	+0.30
	40°	22°	12.30	12.45	−0.15
	30°	10°	14.00	14.00	00
	20°	02°	14.30	14.30	00
Pacific	50°	42°	13.00	12.00	+1.00
	40°	40°	13.00	12.00	+1.00
	30°	28°	13.30	12.30	+1.00
	20°	20°	14.00	13.00	+1.00

the same universal time at conjugate points in opposite hemispheres.

In order to compare critical frequencies at conjugate points at the solstices, *F2* layer daytime temperature differences between summer and winter hemispheres must be taken into account. Kallmann (1961) has deduced from satellite data that temperatures in the *F2* layer of the ionosphere may vary from $\sim 1000°K$ at night to $\sim 2000°K$ by day. This implies that the temperature of the ionosphere depends strongly upon the amount of solar radiation incident upon it, and that there should be a significant difference between mid-day temperatures in the summer and winter hemispheres. We have made the simple assumption that

$$\left(\frac{T_S}{T_N}\right)_{\text{daytime}}=\left(\frac{\cos\chi_S}{\cos\chi_N}\right)_{\text{noon}}=\frac{\cos(\lambda_S\pm\delta)}{\cos(\lambda_N\mp\delta)} \quad (5)$$

where

χ_N and χ_S are the solar zenith angles at the geographic latitudes λ_N and λ_S of a pair of conjugate points, and δ is the solar declination. (This assumes in effect that solar electromagnetic radiation is the only source of heating and that heat loss is by constant downward conduction.)

Then, from equations (4) and (5)

$$(f_0F2)_N\sqrt[4]{\cos(\lambda_N\pm\delta)}=(f_0F2)_S\sqrt[4]{\cos(\lambda_S\mp\delta)} \quad (6)$$

Equation (6) shows that daytime critical frequencies should be higher in the winter than in the summer hemisphere, and this is in fact a well known effect.

Figs. 3 (a) and (b) show monthly mean values of

$$(f_0F2)_N\sqrt[4]{\cos(\lambda_N\pm\delta)}$$

plotted against

$$(f_0F2)_S\sqrt[4]{\cos(\lambda_S\mp\delta)}$$

for several pairs of conjugate points in the four ionospheric zones for December, 1960, and July, 1961. In view of the crude way in which temperature differences between hemispheres have been taken into account, there is remarkably good agreement between critical frequencies at conjugate points at the solstices. However, in the western zone in July critical frequencies (temperature corrected) in the southern hemisphere are considerably lower than in the northern hemisphere. This may be because corpuscular

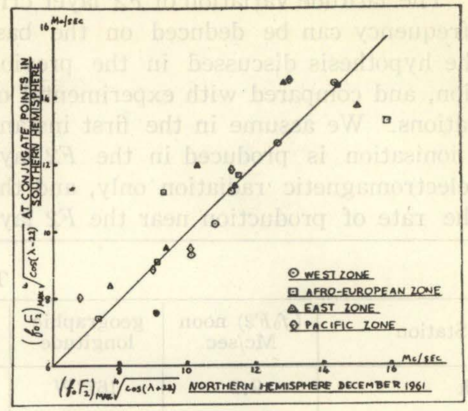

Fig. 3(a).

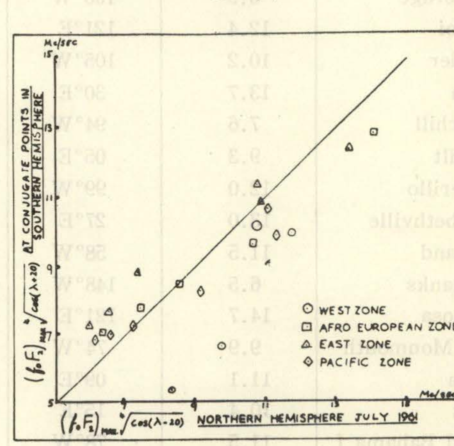

Fig. 3(b).

heating of the ionosphere from particles lost from the inner Van Allen zone becomes important in the region of low magnetic field in South America in local winter. (The rate of loss of these particles into the atmosphere would not be constant, but would reach a maximum near the middle of the day when the upper atmosphere has the greatest density at a given height.) Since

$$(f_0F2)\propto\sqrt{\frac{\phi}{\sqrt{T}}}$$

it follows that higher *F2* layer temperatures give lower values of critical frequency for a given equilibrium flux ϕ of ions. In fact, since the average value of total magnetic field is higher in the northern than in the southern hemisphere, there should be relatively more corpuscular heating of the ionosphere, and consequently lower average critical frequencies, in the southern than in the northern hemisphere.

4. The latitude variation of *F2* layer critical frequency can be deduced on the basis of the hypothesis discussed in the previous section, and compared with experimental observations. We assume in the first instance that ionisation is produced in the *F2* layer by electromagnetic radiation only, and that q, the rate of production near the *F2* layer maximum, is proportional to the cosine of the solar zenith angle χ, for $\chi < 85°$. We further assume that the ion loss rate in the *F2* layer is proportional to the ion density (Ratcliffe, [1956]) and that there is an approximate equilibrium between combined production and loss of particles in the *F2* layers at two conjugate points over twenty-four hours.

Table II.

Station	(f_0F2) noon Mc/sec	geographic longitude	geographic latitude	conjugate latitude	'average' latitude	geomagnetic latitude
Adak	9.9	167°W	52°N	42°S	47°	48°
Akita	12.2	140°E	40°N	22°S	31°	30°
Anchorage	6.9	150°W	61°N	52°S	57°	61°
Bagnoi	12.4	121°E	16°N	05°N	06°	05°
Boulder	10.2	105°W	40°N	56°S	48°	49°
Bunia	13.7	30°E	02°N	17°N	07°	00°
Churchill	7.6	94°W	59°N	77°S	68°	69°
De Bilt	9.3	05°E	52°N	45°S	49°	52°
El Cerillo	12.0	99°W	19°N	37°S	28°	29°
Elizabethville	13.0	27°E	12°S	30°N	−21°	−14°
Falkland	11.5	58°W	52°S	22°N	−37°	−41°
Fairbanks	6.5	148°W	65°N	58°S	62°	63°
Formosa	14.7	121°E	25°N	06°S	16°	14°
Fort Monmouth	9.9	74°W	40°N	68°S	54°	51°
Genoa	11.1	09°E	45°N	31°S	38°	46°
Gratz	10.4	15°E	47°N	32°S	40°	47°
Grand Bahama I.	11.5	78°W	27°N	50°S	39°	38°
Huancayo	12.6	75°W	12°S	13°S	00°	−01°
Inverness	8.3	04°W	57°N	58°S	57°	60°
Kiruna	8.2	20°E	68°N	60°S	64°	65°
Leopoldville	14.0	15°E	04°S	25°N	−10°	−03°
Lulea	8.4	22°E	66°N	54°S	60°	63°
Maui	13.2	156°W	22°N	20°S	20°	20°
Nurmijarvi	8.8	25°E	60°N	52°S	56°	57°
Ottawa	9.8	76°W	45°N	74°S	60°	56°
Point Barrow	6.0	157°W	71°N	58°S	65°	68°
Rome	11.2	12°E	42°N	29°S	36°	42°
St. Johns	9.7	53°W	48°N	78°S	63°	59°
Singapore	12.7	104°E	01°N	15°N	−07°	−10°
Slough	9.2	00	52°N	42°S	47°	55°
Sodankyla	8.2	27°E	67°N	53°S	60°	64°
Talara	13.1	81°W	05°S	16°S	05°	06°
Tokyo	12.4	139°E	36°N	18°S	27°	26°
Tromso	8.5	19°E	70°N	61°S	65°	67°
Upsala	8.9	18°E	60°N	51°S	56°	58°
Wakkanai	11.5	142°E	45°N	26°S	36°	35°
White Sands	11.4	106°W	32°N	45°S	38°	41°
Winnipeg	7.8	97°W	50°N	68°S	59°	59°
Yamagawa	12.9	130°E	31°N	12°S	22°	20°

The average number of ions $\bar{N}$/cm.³ produced in twenty-four hours is proportional to $(\cos\chi)_{\text{noon}}$, so we can write

$$C(\cos\chi)_{\text{noon}}+C(\cos\chi_{\text{conj.}})_{\text{noon}}=K(\bar{N}+\bar{N}_{\text{conj.}}) \tag{7}$$

where

C is constant over a day, but varies over the sunspot cycle and has its largest value at sunspot maximum, and K is the loss coefficient.

From equation (4)

$$\frac{N}{N_{\text{conj.}}}=\sqrt{\frac{T_{\text{conj.}}}{T}}$$

so

$$\bar{N}=C\left[\frac{\cos(\lambda\pm\delta)+\cos(\lambda_{\text{conj.}}\mp\delta)}{K(1+\sqrt{T/T_{\text{conj.}}})}\right] \tag{8}$$

At the equinoxes $\delta=0$, and we can assume that $\frac{T}{T_{\text{conj.}}}\simeq 1$, so

$$N=\frac{C}{2K}\left[(\cos\lambda+\cos\lambda_{\text{conj.}})\right] \tag{9}$$

The expression for equionoctial noon critical frequencies can therefore be written in terms of the latitudes of the two conjugate points, if

$$(f_0F2)_{\text{noon}}\propto\sqrt{\bar{N}}$$

i.e.

$$(f_0F2)_{\text{noon}}=C'\left\{\frac{C}{K}\left[\cos\left(\frac{\lambda+\lambda_{\text{con.}}}{2}\right)\cdot\cos\left(\frac{\lambda-\lambda_{\text{conj.}}}{2}\right)\right]\right\}^{1/2} \tag{10}$$

If we define the 'average latitude' λ_{av} of two conjugate points as $\frac{1}{2}(\lambda+\lambda_{\text{conj.}})$, then since $\cos\frac{1}{2}(\lambda-\lambda_{\text{conj.}})\simeq 1$,

$$(f_0F2)_{\text{noon}}=C'\left\{\frac{C}{K}\left[(\cos\lambda_{\text{av}})\right]\right\}^{1/2} \tag{11}$$

Table 2 lists the noon critical frequencies (monthly mean values) of a number of stations around the world, for March, 1960, together with their geographic positions, their geomagnetic latitude 1, the approximate geographic latitude of their conjugate points, and the average geographic latitude of each conjugate pair. It can be seen that λ_{av} is nearly equal to 1 for each station (this relation would be exact if the earth's field were a simple dipole). A rough estimate of the geographic latitude of the conjugate point of any station can, therefore, be made from a knowledge of its own geographic and geomagnetic latitudes for

$$\lambda_{\text{conj.}}=2L-\lambda \tag{12}$$

In Fig. 4 (a), $(f_0F2)_{\text{noon}}$ has been plotted against $(\cos\lambda_{\text{av}})^{1/2}$ for the stations listed in Table 2. There is a straight line relation between $(f_0F2)_{\text{noon}}$ and $(\cos\lambda_{\text{av}})^{1/2}$ at middle latitudes, but there are systematic departures from it at high latitudes and also near the equator. Fig. 4 (b) shows, for comparison, $(f_0F2)_{\text{noon}}$ plotted against $(\cos\lambda)^{1/2}$ (the geographic latitude of the station only).

In this case there is a wide scatter of points at all latitudes. The relatively high values of critical frequency in the auroral zones and under the outer radiation belt in Fig. 4 (a) may be due to additional ion production by corpuscular radiation. However, there may not be equilibrium between conjugate ionospheres at latitudes above 60° (see equation 1), and so equation (11) may not be valid there. The drop in critical frequencies near the magnetic equator may arise because the value of the loss coefficient K, appropriate to lines of force at low latitudes is greater than at higher latitudes.

5. Experimental observations of daytime $F2$ layer critical frequencies suggest strongly that the $F2$ layers of the ionospheres at mag-

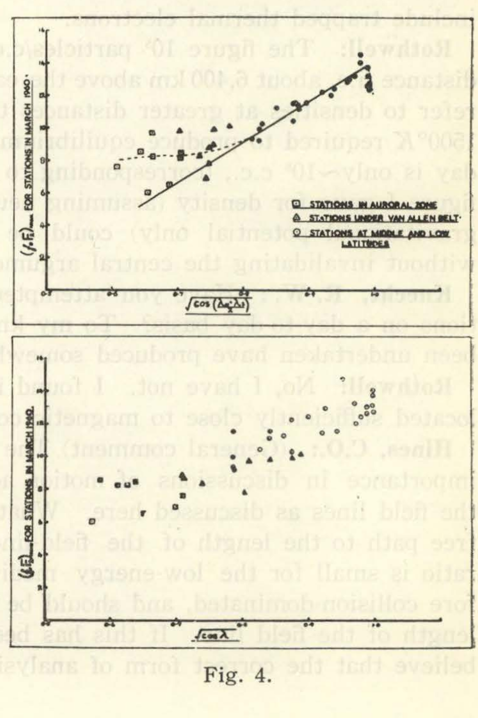

Fig. 4.

netic conjugate points are inter-related, and give support to the hypothesis that large fluxes of ionospheric protons and electrons travel along lines of force between hemispheres.

It is therefore quite possible that the ionosphere is an important source of trapped electrons in the radiation belts. It is known from satellite observations that during magnetic storms trapped electrons in the outer Van Allen belt undergo changes in pitch angle and may be accelerated to energies greater than their pre-storm value. (Fan *et al.* [1961], Van Allen and Lin [1960], Arnoldy *et al.* [1960]) If the same processes affect ionospheric electrons, changes in pitch angle near the equatorial plane could result in trapping of electrons which would otherwise travel only from one ionosphere to the conjugate ionosphere without magnetic reflection.

Acknowledgement

The author would like to thank Professor J. Sayers for much advice and helpful criticism.

References

Arnoldy, R. L., Hoffman, R. A. and Winckler, J. R.: J. Geophys. Research **65** (1960) 3004.

Bates, D. R. and Massey, J. W. S.: Proc. Roy. Soc. Soc. **187** (1946) 261.

Fan, C. Y., Meyer, P. and Simpson, J. A.: Second Space Science Symposium, Florence (1961)

Johnson, F. S.: J. Geophys. Research, **65** (1960) 2571.

Kallman, H. K.: J. Geophys. Research, **66** (1961) 787.

Ratcliffe, J. A.: J. At. Terr. Phys. **8** (1956) 260.

Setty, C. S. G. K.: J. At. Terr. Phys. **19** (1960) 88.

Van Allen, J. A. and Lin, W. C.: J. Geophys. Research, **65** (1960) 2998.

Vestine, E. H. and Sibley, W. L.: J. Geophys. Research, **65** (1960) 1967.

Discussion

Ratcliffe, J. A.: Why does the author not mention the conclusion, of those who study satellite orbits, to the effect that there is no measurable difference of temperature between the northern and southern hemispheres in the upper atmosphere?

Rothwell, P.: It would be difficult to detect the seasonal variation in the maximum of the diurnal temperature variation from satellite drug measurements. I believe that direct rocket measurements of temperature at high altitude do indicate a seasonal change.

Dungey, J. W.: You mentioned a plasma density$\sim 10^5\,\mathrm{cm}^{-3}$ far out. Surely you should consider the whistler measurements. Note that these measurements would include trapped thermal electrons.

Rothwell: The figure 10^5 particles/c.c. was given for two earth radii geocentric distance (i.e. about 6,400 km above the earth) whereas I believe whistler measurements refer to densities at greater distances than this. However, the fluxes of particles at$\sim$ $1500°K$ required to produce equilibrium between ionospheres during the course of a day is only$\sim 10^8$ c.c., (corresponding to densities of 100–1000 particles c.c.) so that the figure I gave for density (assuming neutral matter was distributed according to the gravitational potential only) could be reduced by two or three orders of magnitude without invalidating the central argument.

Knecht, R. W.: Have you attempted a correlation of f_0F2 at conjugate point stations on a day-to-day basis? To my knowledge those studies of this type that have been undertaken have produced somewhat variable results.

Rothwell: No, I have not. I found it difficult to find ionosphere stations that were located sufficiently close to magnetic conjugate points.

Hines, C.O.: (General comment) The ratio of ion gyro to collision frequency is of importance in discussions of motion across the field lines, but not of motions along the field lines as discussed here. What is important here is the ratio of ion mean free path to the length of the field-line from one hemisphere to the other, and this ratio is small for the low-energy medium under discussion. The process is therefore collision-dominated, and should be treated as a diffusion process along the whole length of the field line. If this has been done—and I am not clear on that—I would believe that the correct form of analysis had been adopted.