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II-1C-5. Amplification of the VLF Electromagnetic Wave by a Proton Beam through the Exosphere

Ken-ichi MAEDA and Iwane KIMURA
Department of Electronics, Kyoto University, Kyoto, Japan

A possibility is examined of amplification of a VLF electromagnetic wave by means of a charged particle beam running through the exosphere, with a view to applying it to the originating mechanism of the VLF emissions.

Traveling wave tube (TWT) like mechanism was one of the hypotheses explaining the VLF emissions. However it seems doubtful that such a mechanism can really occur in the exosphere.

In place of TWT mechanism, it is found that an interaction of the VLF wave with a proton cyclotron wave of a beam, works as an amplifying mechanism, although the gain of amplification is not so great, say 20 db per 1000 km coupling. Therefore a powerful noise seed must be needed, in order to interpret the VLF emissions. This noise would come from the vicinity of the ground through the ionosphere by the whistler mode.

§ 1. Introduction

Recently various VLF phenomena spontaneously originated in the exosphere named VLF emissions, have drawn our attention as well as the whistlers do. It is because they have been thought to be generated in the exospheric space and it seems possible for us to bring out some meaningful information of the physical properties of the space by analysing the observed VLF emissions.

An important feature of the VLF emissions is the frequency time characteristics especially for the discrete type of the emissions, such as dawn chorus, risers, hooks etc. And the observed maximum intensity is as large as 10^{-13} w/m^2(c/s) in power flux density.

Several hypotheses have been presented so far, as listed in Table I. These hypotheses were first proposed in order to explain the frequency time characteristics of the discrete type of emissions. The interpretation relies on the condition of amplification or of emission taking place, that is the phase velocity (v_{ph}) to be nearly identical with the beam speed U_0, $v_{ph}\simeq U_0$, as firstly given by Gallet and Helliwell.[1) 2)]

In regard to the theoretically expected intensity of emission, Cherenkov radiation looks most effective, but still insufficient to give the observed one. Then we have to suppose that an amplification mechanism like TWT will cooperate to bring about the observable emissions, in addition to the radiation mechanism. In this respect, the TWT mechanism proposed by Gallet and Helliwell might have quite an important rôle for the VLF emissions.

Although Bell and Helliwell[3)] reported the possible gain of amplification to be 2 db per wave length by the TWT mechanism, the possibility seems doubtful. The purpose of this paper is first to re-examine the TWT theory applied to the incoming charged particles through a magneto-active plasma like the exosphere and to investigate if an alternative mechanism of amplification is possible. The

Table I. Hypotheses of the originating mechanism.

	Condition	Possible gain or power flux density	Remarks
Traveling wave tube like mechanism	$v_{ph}\simeq U_0$	2 db per wave length	Gallet[1),2)], Helliwell[2),3)], Bell[3)]
Cherenkov radiation	$v_{ph}=U_0\cos\theta$	10^{-20} w/m^2(c/s)	Ellis[4)]
Doppler-shifted cyclotron radiation	$f=f_p/(1-U_0/v_{ph})\rightarrow v_{ph}\simeq U_0$	10^{-21} w/m^2	MacArthur[5)]

second purpose is to find a favorably intense seed for the amplification if a certain mechanism is possible in the exosphere.

§ 2. Fundamental equations and assumptions

The case which will be treated is that a charged particle (electrons and protons) beam runs through an ambient plasma, where a static magnetic field permeates. The beam can be assumed to flow on the average along lines of the static magnetic field, neglecting a drift perpendicular to the geomagnetic line of force. It is also assumed that the beam and the ambient plasma are uniform over a sufficiently large region of interest.

In order to determine the propagation modes of VLF electromagnetic waves in such a coexisting region of the ambient plasma and the beam, the following equations are necessary and sufficient.

1. Maxwell's field equations.
2. The equations of conservation of electrons and protons constructing the beam.
3. Maxwell's laws of the transfer of momentum in mixtures of different kinds of particles.

In applying the above equations to our case, further assumptions will be made. 1. Small signal theory. 2. Plane wave solution; we suppose the solution of the equations to be a form $e^{j(\omega t-k_0 nz)}$ where k_0 is the propagation constant in free space, and n is the refractive index in the medium. 3. Zero temperature plasma and beam. 4. Collisionless media.

§ 3. Dispersion equation and coupling mode

By using the above fundamental equations and assumptions, we have a dispersion equation which determines all possible modes of wave in the medium, including not only electromagnetic wave but also electric wave.

In the TWT case we pay no regard to any motion of the protons. Let us consider that the direction of the wave propagation is in the z direction of the Cartesian coordinates and makes an angle θ with the direction of the constant magnetic field, that is, of the beam flow.

When $\theta=0$, we obtain the dispersion equation which determines separately transverse (electromagnetic wave) mode and longitudinal (electric or space charge wave) mode. The latter mode becomes itself complex whenever the wave frequency is less than the electron plasma frequency of the ambient plasma. That is to say, the longitudinal space charge wave is itself spatially growing or decaying, without any interaction with the electromagnetic wave (say, whistler mode wave).

TWT, in principle, relies on the interaction between the above longitudinal space charge wave and also longitudinal circuit wave. In the exosphere, however, for the TWT like amplification to take place, θ must not be zero, because if $\theta=0$ there is no longitudinal component of the electromagnetic wave. Then we calculated the case where $\theta\neq 0$. But it was found out that even in this case the whistler wave mode does not couple with space charge wave mode, so that the whistler mode wave is not always amplified under the influence of the beam.

On the other hand we have noticed that the protons in the beam seems to have strong effect when Doppler shifted proton gyrofrequency coincides with the frequency of the electromagnetic wave of interest. In this case, for convenience, we will treat only transverse case ($\theta=0$). From the dispersion equation which is derived from the fundamental equations and assumptions previously mentioned, we have the following coupling equation;

$$(n-n_0)(n-n_1)+X_p Y_p/2n_1 V_p=0\,, \qquad (1)$$

in case $n\simeq n_0\simeq n_1$, where $X_p=f_p^2/f^2$, $Y_p=f_{Hp}/f$, f_p being the proton plasma frequency of the beam, f_{Hp} the proton gyrofrequency, f the wave frequency and $V_p=U_p/c$, U_p being the proton beam velocity and c being the light velocity. And also in this equation, n_0 is the refractive index for the whistler mode and n_1 is the refractive index of the proton slow cyclotron mode, that is,

$$n_0=\sqrt{1-X_p-X_a/(1-Y_e)},\ n_1=(1+Y_p)/V_p\,, \qquad (2)$$

where $X_a=f_a^2/f^2$, $Y_e=f_{He}/f$, f_a being the electron plasma frequency of the ambient plasma, and f_{He} the electron gyrofrequency. We can, thus, understand that the eq. (1) indicates the interaction between the whistler mode and proton slow cyclotron mode.

The roots n of (1) are given by

$$n=\tfrac{1}{2}(n_0+n_1)\pm\sqrt{\tfrac{1}{4}(n_0-n_1)^2-X_p Y_p/2n_1 V_p}\,. \qquad (3)$$

The roots become complex only when

$$\tfrac{1}{4}(n_0-n_1)^2 < X_p Y_p / 2n_1 V_p \quad (4)$$

and the imaginary part of n will be maximum if

$$n_0 = n_1\,, \quad (5)$$

then

$$n = n_1 \pm j\gamma,\ \gamma = \sqrt{X_p Y_p / 2n_1 V_p} \simeq \sqrt{X_p Y_p / 2} \quad (6)$$

These complex roots may imply that the whistler mode wave and proton cyclotron wave simultaneously grow or decay as they propagate in the positive z direction. The gain Γ of the amplification (or the rate of growing) due to the coupling length l, will be given by $\Gamma = e^{k_0 \gamma l}$. For example, assume $N_p = 10^7\ \mathrm{m}^{-3}$, $f_{Hp}/f = 5.45 \times 10^{-2}$ (or $f_{He}/f = 10^2$) and $l = 10^6$ m, then $\Gamma = e^{2.3} = 20$ db. Fig. 1 shows how the gain Γ depends upon the proton density N_p of the beam.

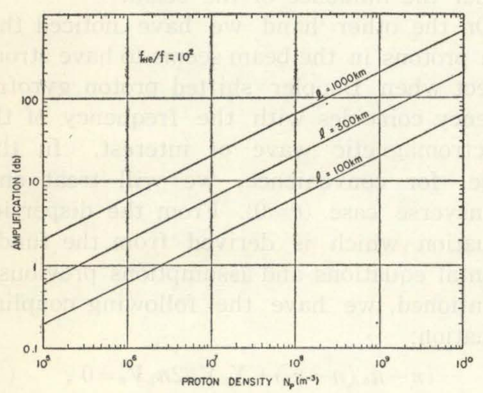

Fig. 1. Amplitude gain (Γ) of amplification versus proton density of the beam.

(5) is a necessary condition of the amplification and alternatively represented as $v_{ph} = U_p/(1+Y_p) \simeq U_p$, because Y_p is much smaller than unity. This condition is clearly identical with that given by Gallet and Helliwell in their TWT theory.[1) 2)]

The frequency which satisfies (4) prescribes the frequency band to be simultaneously amplified, by a single velocity of the beam. This band is approximately given by $\Delta f = 8\gamma f/n_1$.

The energy required for amplifying both the whistler mode and the proton cyclotron mode is supplied from the kinetic energy of the proton beam velocity. As the result, the beam speed is slowed down according as the above two mode waves grow up.

§ 4. Interpretation of the VLF emissions

As was described in § 1 the true mechanism of the VLF emissions should enable us to explain not only the frequency time characteristics but also the observed intensity. As for the former, the proton amplification mechanism is satisfactory because the condition of amplification is just the same as that of the TWT. As for the latter, the intensity, our hypothesis of amplification needs a certain noise seed to be amplified. If Cherenkov radiation be the seed, more than 80 db of the amplitude gain would be required. Consequently we are urged to suppose either much more density than $10^8\mathrm{m}^{-3}$ of the proton beam according to Fig. 1 or some powerful background noise in the exosphere. Such a high density may be possible in case of a strong magnetic storm, but may be of rare chance.

Now we consider the latter, strong background noise. A possibility will be ascribed to the whistler mode of propagation in the ionosphere. In other words, if there exists a certain intense noise in the space under the ionosphere, the exosphere will also be filled with this same noise on account of the transparency of the ionosphere for VLF region due to the whistler mode.

According to the world wide contour maps of noise intensity on the ground published by C.C.I.R.[6)] in 1959, the electric field intensity of about 3 kc/s on the ground is as strong as $10^2 \sim 10^3\ \mu$v/m for 1 kc/s of band width.

Taking into account the absorption of the whistler mode wave within the ionosphere (about 5 db during nighttime) the power flux density of the noise in the exosphere at night would become larger than 10^{-15} w/m²(c/s). This intense noise with a continuous spectrum, needs only about 20 db of amplification to be heard as the VLF emissions on the ground.

References

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Discussion

A (Unidentified person): In your Table I showing the estimated intensity of the emissions, you gave the power density of 10^{-20}w/m^2(c/s) for Cherenkov radiation, which was calculated from the emission of individual particles. If you take into account the stimulated emission, you will get much larger power. The approach is to take into account the things you have rejected in your treatment.

Dungey, J. W. (to A): You mean the stimulated emission corresponds to the amplification?

A (to Dungey): Yes, that's really what I mean.

Kimura, I.: It may be an alternative interpretation of the amplification mechanism. My way of thinking is that our proton mechanism is a kind of "anomalous Doppler effect" which was first reported theoretically by V. L. Ginzburg (Soviet Physics USPEKHI 2 (1960) 874–893).

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II-1C-P1. Whistlers and VLF Emissions in Connection with the Earth Storms

Ken-ichi MAEDA

Department of Electronics, Kyoto University, Kyoto, Japan

Fundamental features of whistlers and VLF emissions including hiss, dawn chorus and others are briefly reviewed and important characteristics in disturbed states, which have been reported by the papers presented in this symposium as well as by other papers, are studied. The related theories to interpret the phenomena are surveyed, and discussions are given on some important characteristics of the phenomena. Some suggestions on data reduction and on possible experimental studies by means of satellite are also proposed.

§1. Fundamental features

The very low frequency (VLF) radio phenomena which are treated in this paper contain a series of natural radio noises at frequencies between 1 to 30 kc, known as "Whistlers," "Hiss," "Dawn Chorus" and similar rising and falling tones. For the sake of convenience whole phenomena are classified into three groups. The first group (W) includes the whistlers, the second group (C) includes the dawn chorus, hooks, quasi-horizontal and falling tones and the third group (H) includes the hiss, which is called "Noise burst" or "Noise Storm" by Ellis and others. The second group in this paper is conventionally represented by "Chorus."

Following is a summary report on fundamental features of the VLF phenomena, which have been revealed mainly by systematic observations carried out in the U.S.A. and in Australia and New Zealand.

1) Time duration. This is short in case of W and C, say, the order of 0.1—1 second, while that of H is long, say, the order of