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II-5-14. Intensity-Time Relationship of Cosmic Ray Unusual Increase, not Characterized by Impact Zone Effects

Kazuo NAGASHIMA

Physical Institute, Nagoya University, Nagoya, Japan

Under an assumption that the solar-produced cosmic rays are able to reach the earth only after the diffusion process through the interplanetary space, the intensity-time relationship at an observed point is studied theoretically. By comparison of the observed intensity-time relationship with the theoretical one, it is possible to determine the initial time and the injection period of cosmic rays which are injected into interplanetary space from the source, to determine the diffusion constant with which the cosmic rays propagate in the space and to estimate the total number of cosmic rays injected during the injection period.

We assume that at a time of unusual increase the interplanetary space of the solar system is uniformly disturbed and the cosmic rays injected into the space from the sun propagate through it by the diffusion process with a constant mean free path which is very small compared with the distance between the sun and the earth.

Other factors necessary for solving the diffusion equation are the source function S and the boundary conditions. The source function is an injection rate of cosmic ray particles from a source and is a function of cosmic ray rigidity P and time t. At the present time, there is only a little information about the rigidity and time dependences of S. And so, we assume that S can be approximated by one of the following types,

$$\begin{aligned} &(\mathrm{I}) \quad S=C\delta(r)\delta(P-P_0)\delta(t) \\ &(\mathrm{II}) \quad S=C\delta(r)f(P)\delta(t) \qquad\qquad (1) \\ &(\mathrm{III}) \quad S=C\delta(r)\begin{Bmatrix}\delta(P-P_0)\\ f(P)\end{Bmatrix}u(t)u(-t+t_s)\,, \end{aligned}$$

where δ is the delta function and u is the step function defined by

$$\left.\begin{aligned} u(x)&=1 \quad \text{for} \quad x\geq 0 \\ &=0 \quad \text{for} \quad x<0\,. \end{aligned}\right\} \qquad (2)$$

Type (I) represents an instantaneous injection of cosmic rays of monochromatic rigidity P_0, Type (II) represents an instantaneous injection of cosmic rays of a continuous momentum spectrum and Type (III) a continuous and constant injection of cosmic rays of monochromatic or continuous momentum spectrum during a time interval, Δt_s. Hereafter, Δt_s will be refered to as the injection period.

As the boundary condition, we assume that the diffusion region extends to a distance, r_b, measured from the source and the mean free path is infinite outside this region. We define ξ as the ratio of r_0 to r_b, where r_0 is the distance of the observed point from the source.

$$\xi = r_0/r_b \qquad (3)$$

The purpose of the present analysis is to get some information about mean free path, source function and boundary condition from the intensity-time relationship at an observed point. For this purpose, the following normalization is applied. As shown in Fig. 1, the instantaneous time, t, is measured from the time of maximum intensity, t_M, and expressed in units of the half-time, $\Delta t_{+1/2}$, during which the maximum intensity recovers to its half value. Hereafter, this normalized time is refered as τ. The intensity is nor-

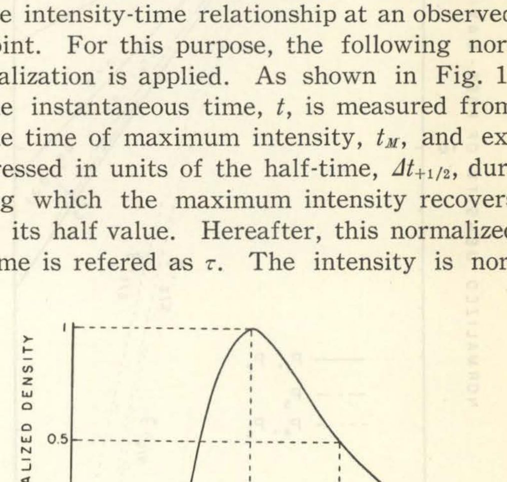

Fig. 1. Normalization of the intensity of cosmic rays.
t_S: injection period,
t_M: time of maximum intensity,
$\Delta t_{+1/2}$: half time.

malized by the maximum intensity. We shall refer the period of the increasing portion of the intensity as the pre-maximum period and the period of the decreasing portion as the post-maximum period (cf. Fig. 1).

For a case when the diffusion region extends to infinity, the normalized intensity-time relationship is shown in Figs. 2 and 3 for the post- and pre-maximum periods, respectively. The curve attached by $n=3$ shows the intensity for the source function of Type (I). For comparison, curves attached by $n=2$ and 1 are shown which correspond respectively to two dimensional and one dimensional diffusions for the source function of Type (I). In these cases, the source may be regarded as being in the forms of an infinite line and of an infinite plane. It might be

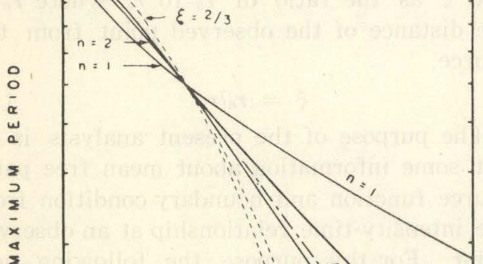

Fig. 2. Normalized intensity-time relationship during the post-maximum period.

Full lines are for the source function Type (I) or (II), broken lines for that of Type (III) and dotted lines for that of Type (I) with a finite extension of the boundary.

needless to say that these normalized curves are independent of the diffusion constant during all the period. In the following, only three dimensional diffusion is considered.

We shall consider next the case that the source function is expressed by Type (II), which represents the instantaneous injection of cosmic rays of continuous rigidity spectrum. If cosmic ray particles are assumed

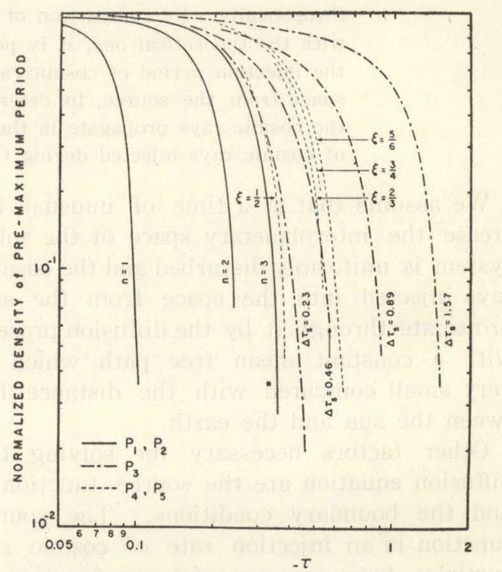

Fig. 3. Normalized intensity-time relationship during the pre-maximum period.

Full lines are for the source function Type (I) or(II), broken lines for that of Type (III) and dotted lines for that of Type (I) with a finite extension of the boundary.

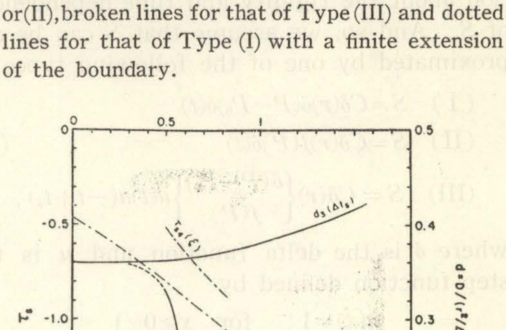

Fig. 4. Diffusion constant, D, and normalized initial time, τ_S, as functions of $\Delta\tau_S$, the normalized injection period.

to be injected with a power spectrum of rigidity, propagate through medium with diffusion constant proportional to a power of rigidity, and produce the secondary cosmic rays in the earth's atmosphere with a yield function of a power spectrum of rigidity, then it is possible to prove that the normalized intensity observed in the atmosphere is exactly the same as that produced by Type (I). Therefore, the source function of Type (II) is equivalent to that of Type (I), within the limitation mentioned above.

The only factor which influences the shape of the intensity-time relationship is the injection period, Δt_s. For the pre-maximum period, (cf. Fig. 3) broken lines represent the intensities characterized by different values of the injection period for point source. On the other hand, for the post-maximum period, (cf. Fig. 2) there are only slight changes from the intensity produced by Type (I) or (II).

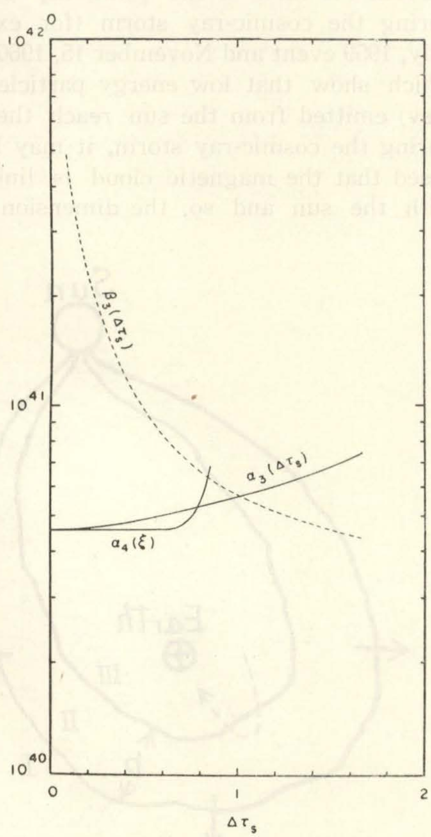

Fig. 5. Total number, N, of cosmic rays injected during the injection period, $\Delta\tau_s$.

Consequently, a comparison of the normalized observed curve with the theoretical one provides a determination of the value of $\Delta\tau_s$, the normalized injection period. The normalized initial time of injection τ_s, is determined as a function of $\Delta\tau_s$ (cf. a broken line attached by $\tau_{s3}(\Delta\tau_s)$ in Fig. 4). The diffusion constant, D, is given by

$$D = d(r^2/\Delta t_{+1/2}), \tag{4}$$

where r is the distance of the observed point from the source and d is a function of $\Delta\tau_s$ shown by the full line attached by $d_3(\Delta\tau_s)$ in Fig. 4. From Fig. 4, we see that D is almost independent of $\Delta\tau_s$. Therefore, it is possible to estimate the value of D within a several factor even without any knowledge about the injection period.

The total number, N, of cosmic rays injected during the injection period is determined by the values of $\Delta\tau_s$ and the cosmic ray density, ρ_M, observed at the time of maximum (see the full line attached by $\alpha_3(\Delta\tau_s)$ in Fig. 5). This quantity also is almost independent of $\Delta\tau_s$.

The existence of a finite boundary of the diffusion region complicates matters in the intensity-time relationsphip. Dotted lines in Figs. 2 and 3 represent the intensities characterized by different values of ξ for the source function of Type (I).

For the pre-maximum period (cf. Fig. 3), the deviation of the curve ($\xi=1/2$) from that of $n=3$ is very small and can be neglected. Therefore, if we could know that the value of ξ is less than about 1/2, it is possible to neglect the influence of the finite extent of boundary for the determination of the value of $\Delta\tau_s$. A method for estimating the value of ξ is provided from Fig. 2. The corresponding deviation from the curve ($n=3$) due to the existence of finite boundary is remarkable even if the value of ξ is about 1/2. Hence, it is easy to determine whether the value of ξ is less than 1/2 or not.

In conclusion, if the distance of the boundary from the source is about two times larger than that of the observed point from the source, it is possible to neglect the influence of the existence of boundary for the determination of quantities mentioned above.