III-3. Origin

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Date	Time	Paper Numbers
Sept. 11	15 : 30 – 17 : 30	from III-3-1 to III-3-8
Sept. 13	11 : 30 – 13 : 30	from III-3-9 to III-3-10

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III-3-1. On the Mechanism of Stellar Explosion

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We describe a method of treating the propagation of shock waves in the stellar interior which we have developed. The problem of stellar explosion has been treated by several authors. Especially, Colgate and others calculated the dynamical process of supernova using electronic computer. Compared to their methods, ours are rather rough approximation. However, when applied under appropriate conditions, it has the advantage of having clear physical image and easiness of calculation.

The method consists of dividing the inhomogeneous layer into infinitesimal ones and applying the law of reflection and refraction of a shock wave to each infinitesimal layer.

The resulting equation for the ideal gas is given by

$$\frac{dz}{d\ln r}=\frac{\left(\frac{1}{n+1}+2\sqrt{\frac{1+\lambda^2 z}{z(1+\lambda^2)}}\right)V-\frac{4}{1+\frac{(z-1)(1-\lambda^2)}{\sqrt{(1+\lambda^2 z)z(1+\lambda^2)}}}}{\frac{2}{z-1}-\frac{1}{z+\lambda^2}+\frac{2}{z-1}\sqrt{\frac{1+\lambda^2 z}{z(1+\lambda^2)}}} \qquad (1)$$

where the usual notations are used as follows:

$$r=\text{radial position},\quad n+1=\frac{d\log p}{d\log T},$$

$$V=-\frac{d\log p}{d\log r},\quad \lambda^2=\frac{\gamma-1}{\gamma+1},$$

γ being the ratio of specific heats.

Especially, the shock strength is defined by the pressure ratio $z=p_2/p_1$; p_1 and p_2 being pressures before and behind the shock front. z and the density ratio $y=\rho_1/\rho_2$ are related by the Hugoniot relation

$$y=\frac{1+\lambda^2 z}{z+\lambda^2}\,.$$

The first term of the numerator of (1) represents the "Pressure Growing", that is, the increase of z with the decrease of pressure towards the stellar surface. The second term represents the decrease of z due to the spherical expansion, or "Spherical Damping".

Analogous formulae, though more involved, are obtained for the most general case, that is, if the Hugoniot relation is given by the implicit expression

$$\varphi(z, y)=0.$$

For example, the radiation pressure, magnetic field and nuclear energy generation can be treated using this general expression.

Now, we consider how strong the power should be for the shock wave to occur in the stellar interior. Suppose a shock front is formed with the strength z at the radius r in the initial star. Then the power required is approximately given by

$$\Pi \sim 2.98\times 10^{46}(M'/R')^{5/2} f(x)\varphi(z)\ \text{erg/sec} \tag{2}$$

where

$$M'=M/M_{\odot},\ R'=R/R_{\odot},\ x=r/R$$

and

$$f(x)\sim 0.1 \text{ near the stellar core}$$

and

$$\varphi(z)\sim 10 \text{ for } z\lesssim 100\,.$$

The factor 10^{46} suggests that it needs an enormous power to raise a shock wave in the stellar interior. Further, the factor $(M'/R')^{5/2}$ suggests that shock waves can arise easier in an extended loose star like a giant than in a *tight* star such as the main-sequence. In other words, stellar outbursts can appear in the normal tight star only when an enormous energy liberation occurs within a short time. This fact, in turn, guarantees the stability of normal stars.

As the shock front passes through the star, its strength increases, and therefore the velocity behind it will become higher and higher, until at last it reaches to the escape velocity at the relevant position. This escape velocity is given by the expression

$$u_{\text{esc.}}=6.17\times 10^2\left(\frac{M'}{R'}\right)^{\frac{1}{2}} h(x)\ \text{km/sec}\,. \tag{3}$$

Here, $h(x)$ has the order of magnitude of one or so.

As the first approach to the problem of mass ejection we assume that the ejection of the mass elements starts when the outward gas velocity becomes the escape velocity. Then by Eq. (3) we can see that, due to the factor $(M'/R')^{\frac{1}{2}}$, a tight star can emit the mass of the envelope with higher velocity than a loose star, if it can explode at all.

Now, we apply our method to the mechanism of supernovae and the origin of planetary nebulae.

As a preliminary test of our theory, we consider a pre-supernova model which has 10 solar masses and a tenth of solar radius, and of which the structure is represented by Eddington's model with the molecular weight of two. There are yet two unknown parameters, x_0 and z_0, the initial front position and the initial shock strength, which can not be determined without detailed knowledge of the mechanism of energy liberation. Here

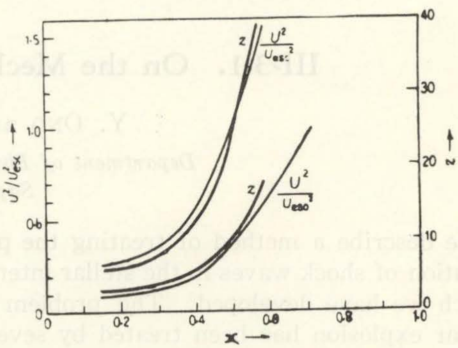

Fig. 1. Shock strength z and the ratio of gas velocity and the escape velocity $(u/u_{\text{esc}})^2$ versus the radius $x=r/R$.

Table I.

Initial shock strength	z_0	3.0	6.0
Initial power	Π_0 erg/sec	9.09×10^{50}	3.34×10^{51}
Mean generated energy	$\bar{\varepsilon}$ erg/g.sec	3.67×10^{17}	1.35×10^{18}
Pulse duration	$t_1=W/\Pi_0$sec	11.0	3.0
Location of separation	r_1/R	0.71	0.49
Thrown mass	ΔM	$\sim 0.1 M_{\odot}$	$\sim 1.2 M_{\odot}$
Escape velocity	u_{esc} km/sec	7200	8800
Propagation time	t_2 sec	~ 5	~ 1.5

$\bar{\varepsilon}\sim\Pi_0/m_0$, Total liberated energy W is assumed as 10^{52} erg.

we tentatively assume that

$$x_0 = 0.141 \ (q_0 = 0.125)$$

and

$$z_0 = 3 \text{ or } 6.$$

The value of x_0 (or q_0) seems to be reasonable from the structure and nuclear data. We have examined which value of z_0 is more reasonable by calculation. The results are shown in Fig. 1 and Table I.

Our main assumption was that pulse duration t_1 is longer than propagation time t_2. The result proves that this assumption is not unreasonable in this case. The fairly good agreements with the more exact results obtained by Colgate and Johnson who used almost the same model, also justifies this assumption. The Table shows that putting $z_0=6$ is better than $z_0=3$, in order to obtain an appreciable amount of thrown mass.

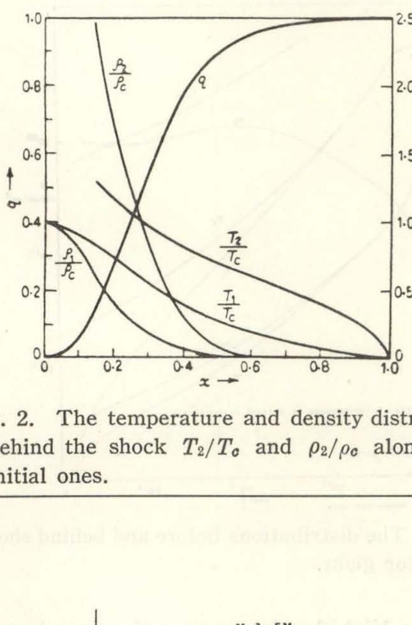

Fig. 2. The temperature and density distribution behind the shock T_2/T_c and ρ_2/ρ_c along with initial ones.

Fig. 2 shows the temperature and density distributions behind the shock front for z_0 of six, along with the initial distributions. Although these distributions would change after the front passes through, they would be maintained fairly constant at least during the passage of the shock pulse, that is, about 3 seconds.

It should be mentioned that the temperature falls very slowly owing to the effect of radiation pressure behind the shock front, while the density falls off rapidly towards the surface. This situation provides a very favorable condition for the rapid nuclear processes to occur in the envelope during the explosion, which are discussed by Hayakawa, Hayashi and others. According to them, the condition for the processes to occur is that the temperature is about 10^9 degrees and the density is about one or ten grams per c.c..

The results shown above seem to reveal that our method is right in principle. Now we are going to apply our method to the actual stars of type I and type II supernovae. We assume that these supernovae have masses of $1.5\,M_\odot$ and $30\,M_\odot$ respectively, and treat the radiation effect in more exact manner. The effect of nuclear reactions or nuclear detonation at the envelope will be also

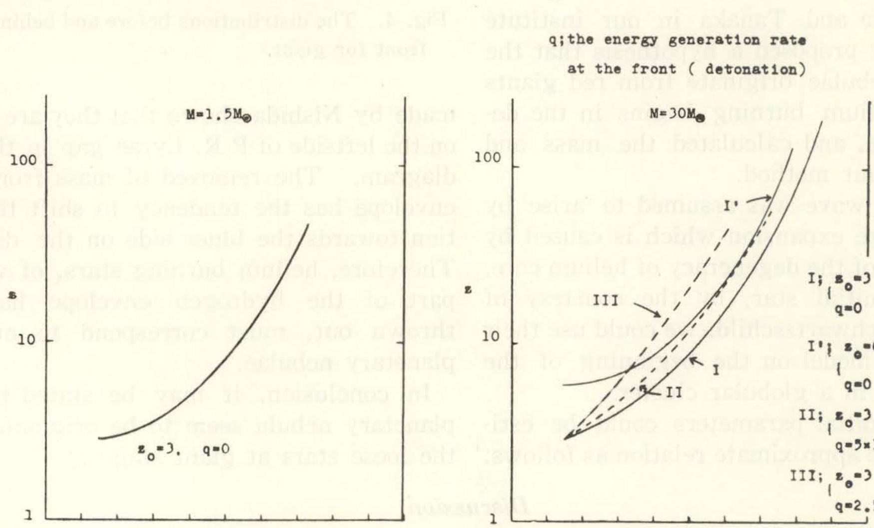

Fig. 3. Shock strength z versus radius $x=r/R$.

considered. Fig. 3 shows the increase of z for several cases. Here, the case II ($q=5\times10^{17}$ ergs/g) corresponds roughly to the surface reactions without neutron source, that is, fusions of light elements C, O and Ne by Fowler and others.

The case III ($q=2.5\times10^{18}$ ergs/g) corresponds roughly to reactions with suitable neutron sources, that is, the rapid C-N-O cycle of hydrogen burning by Hayakawa and others.

We can see that the general features are essentially the same as obtained using the above rough estimation for both types, but the nuclear reactions intensify the shock wave very much.

In order to obtain the light curves and the red shifts of spectral lines observed after the light maximum, we must analyse the expansion stage after the shock reached the stellar surface. To do so, we need the integration of the gas dynamical equations with suitable approximations under appropriate initial conditions.

These calculations are now in progress in our institute with the collaboration of Sakurai and others.

Next, we consider planetary nebulae which are clouds of ionized gases surrounding certain blue subdwarf stars. Their physical state has been investigated very well, but their origin and evolution still remains unsolved.

As an example of loose star emitting masses, Sakashita and Tanaka in our institute have recently proposed a hypothesis that the planetary nebulae originate from red giants when the helium burning begins in the degenerate core, and calculated the mass and velocity by our method.

The shock wave was assumed to arise by the rapid core expansion which is caused by the removal of the degeneracy of helium core. As for the initial star, by the courtesy of Härm and Schwartzschild, we could use their unpublished model on the beginning of the helium flash in a globular cluster.

Also the initial parameters could be estimated by the approximate relation as follows: mean energy generation rate $\bar{\varepsilon}$ is roughly equal to the power of shock wave divided by the mass contained.

Inserting the value $\bar{\varepsilon}=10^{44}$ ergs/sec and other physical values into this relation, the initial strength was estimated as $z_0=1.001$ at the edge of the degenerate core where $q_0=0.39$. Fig. 4 shows the results obtained. The calculated escape velocity is 95 km/sec and the thrown mass is half a solar mass. They agree fairly well with the observed values.

Finally, we would like to mention some reasons to expect that the remnant stars correspond to the nuclei of planetary nebulae. Recent calculation of helium burning models

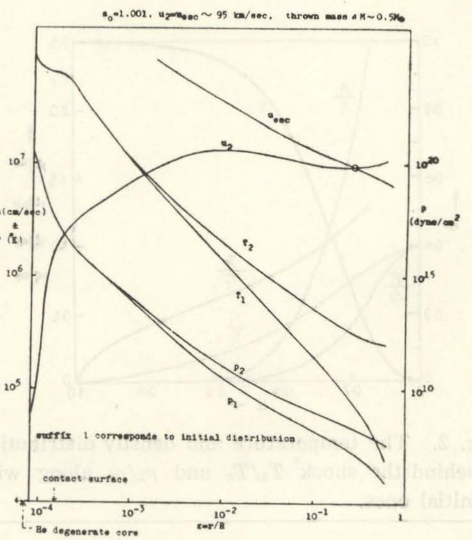

Fig. 4. The distributions before and behind shock front for giant.

made by Nishida shows that they are located on the leftside of R.R. Lyrae gap in the H-R diagram. The removed of mass from outer envelope has the tendency to shift the location towards the bluer side on the diagram. Therefore, helium burning stars, of which a part of the hydrogen envelope has been thrown out, must correspond to nuclei of planetary nebulae.

In conclusion, it may be stated that the planetary nebula seem to be originated from the loose stars at giant stages.

Discussion

Discussion for papers III-3-1 and III-3-2 is combined and given after the Paper III-3-2.