JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN Vol. 17, SUPPLEMENT A-III, 1962
INTERNATIONAL CONFERENCE ON COSMIC RAYS AND THE EARTH STROM Part III

# III-5-3. The Polarization of the High Energy Cosmic Ray Muons*

V. S. BEREZINSKI
*Academy of Science, U.S.S.R.*

The polarization of cosmic ray muons has been calculated in paper (1). The polarization of muons of the energy $E$, produced by the decay of $\pi$-mesons with the Lorentz-factor $\gamma$ in the laboratory system of coordinates, which was found through the relativistic transformation of four-vector spin is

$$\cos\theta=\frac{EE^*}{PP^*}-\frac{\gamma m_\gamma^2}{PP^*} \tag{1}$$

where $E$ and $p$ energy and momentum of muons in the laboratory system,

$$E^*=\frac{m_\pi^2+m_\mu^2}{2m_\pi} \text{ and } P^*=\frac{m_\pi^2-m_\mu^2}{2m_\pi}$$

the energy and momentum of muons in the $\pi$-meson rest system: $m_\mu$ mass of muon at rest; $m_\pi$ mass of $\pi$-meson at rest. There and below the scale of $c=1$ is used. In order to find the average polarization, (1) was averaged over the $\pi$-meson generation spectrum.

But in fact to calculate the average polarization of muon, produced at the given height, it is necessary to average (1) over the function which represents the dependence of the number of muons with the energy $E$, generated at this altitude in the unit of time by $\pi$-mesons of energy $\varepsilon$ by the decay of which the muons under consideration are produced. High accuracy is required to calculate this function, since the polarization is found as the difference between two numbers, each being 3.7. This function is calculated below.

Consider a beam of muons going down vertically, at the energy $E$, produced by the decay of $\pi$-mesons of the energy $\varepsilon$. For the given energies of muons ($2\cdot10^9$–$10^{13}$ ev) the problem can be considered one-dimensional.

The diffusion equation for $\pi$-mesons going down vertically, without taking into account the ionization loss will be

$$\frac{\partial\pi(\varepsilon,x)}{\partial x}=-\kappa(\varepsilon)\pi(\varepsilon,x)+B_0e^{-\beta x}\varepsilon^{-(1+r_\pi)}-\frac{E\pi}{x\varepsilon}\pi(\varepsilon,x) \tag{2}$$

where $x$ is the depth of atmosphere in nuclear units. (The nuclear unit is taken as $80\,\mathrm{gr/cm}^2$[2]). $\pi(\varepsilon, x)$-number of $\pi$-mesons, of the energy $\varepsilon$ at the depth $x$. The first term in the right part of eq. (1) describes the $\pi$-mesons absorption in atmosphere due to nuclear interaction. The coefficient $\kappa(\varepsilon)$ allows for the deflection of the $\pi$-mesons absorption from the exponential law $e^{-x}$. (As a result of nuclear interactions and possibly, in particular, of the regeneration of $\pi$-mesons). The second term gives the production of $\pi$-mesons by nucleon component. $\beta$ is taken as $\beta=0.65$ and $\gamma=1.65$. The third term describes the decay of $\pi$-mesons.

Let us regard the solution of eq. (2) $\pi(\varepsilon, x)$ as a continuous function of the parameter and expand $\pi(\varepsilon, x, \kappa)$ into Taylor series around the value $\kappa=\beta$.

$$\pi(\varepsilon,x,\kappa)=\pi(\varepsilon,x,\mu)+\frac{a}{1!}\frac{\partial\pi(\varepsilon,x,\kappa)}{\partial\kappa}\bigg|+\frac{a^2}{2!}\frac{\partial\pi(\varepsilon,x,\kappa)}{\partial x^2}\bigg|_{\kappa=\mu}+\cdots, \tag{3}$$

where $a=\kappa-\beta$ depends on the $\pi$-meson energy. Substituting (3) in (2) and taking the coefficients before the same power of a as equal, we get

$$\begin{gathered}\frac{\partial\pi(E,x,\beta)}{\partial x}+\left(\beta+\frac{E_\pi}{x\varepsilon}\right)\pi(\varepsilon,x,\beta)=B_0e^{-\beta x}\varepsilon^{-(1+\gamma_\pi)}\\ \frac{\partial}{\partial x}\left(\frac{\partial\pi}{\partial\kappa}\bigg|_{\kappa=\beta}\right)+\left(\beta+\frac{E_\pi}{x\varepsilon}\right)\left(\frac{\partial\pi}{\partial\kappa}\bigg|_{\kappa=\beta}\right)=-1!\,\pi(\varepsilon,x,\beta)\\ \frac{\partial}{\partial x}\left(\frac{\partial^2\pi}{\partial\kappa^2}\bigg|_{\kappa=\beta}\right)+\left(\beta+\frac{E_\pi}{x\varepsilon}\right)\left(\frac{\partial^2\pi}{\partial\kappa^2}\bigg|_{\kappa=\beta}\right)=-2!\left(\frac{\partial\pi}{\partial\kappa}\bigg|_{\kappa=\beta}\right)\end{gathered} \tag{4}$$

* This paper was combined with III-5-2 and presented by A. I. Alikhanyan.

$$\pi(\varepsilon, x)=B_0 e^{-\beta x}\varepsilon^{-(1+\gamma_\pi)}\frac{x}{1+\frac{E_\pi}{\varepsilon}}\left(1-\frac{ax}{2+\frac{E_\pi}{\varepsilon}}+\frac{a^2x^2}{\left(2+\frac{E_\pi}{\varepsilon}\right)\left(3+\frac{E_\pi}{\varepsilon}\right)}-\frac{a^3x^3}{\left(2+\frac{E_\pi}{\varepsilon}\right)\left(3+\frac{E_\pi}{\varepsilon}\right)\left(4+\frac{E_\pi}{\varepsilon}\right)}+\cdots\right), \tag{5}$$

The solution of the first eq. from (4) allowing for the boundary condition $\pi(\varepsilon, x)=0$ at $x=0$

$$\pi(\varepsilon, x, \beta)=B_0 e^{-\beta x}\varepsilon^{-(1+\gamma_\pi)}\frac{x}{1+\frac{E_\pi}{\varepsilon}},$$

that of the second:

$$\left.\frac{\partial\pi}{\partial\kappa}\right|_{\kappa=\beta}=-B_0 e^{-\beta x}\varepsilon^{-(1+\gamma_\pi)}\frac{x^2}{\left(1+\frac{E_\pi}{\varepsilon}\right)\left(1+\frac{E_\pi}{\varepsilon}\right)}.$$

Substituting all these solutions in (3) we have:

$$\pi(\varepsilon, x)=B_0 e^{-\beta x}\varepsilon^{-(1+\gamma_\pi)}\frac{x}{1+\frac{E_\pi}{\varepsilon}}\left(1-\frac{ax}{2+\frac{E_\pi}{\varepsilon}}+\frac{a^2x^2}{\left(2+\frac{E_\pi}{\varepsilon}\right)\left(3+\frac{E_\pi}{\varepsilon}\right)}-\frac{a^3x^3}{\left(2+\frac{E_\pi}{\varepsilon}\right)\left(3+\frac{E_\pi}{\varepsilon}\right)\left(4+\frac{E_\pi}{\varepsilon}\right)}+\cdots\right), \tag{5}$$

In the unit of time at the height $x$, $\varepsilon t_0/m_\pi \pi(\varepsilon, x)d\varepsilon dx$ $\pi$-mesons of the energy $\varepsilon$ decay. $t_0$- is life time of a $\pi$-mesons in the rest frame. By the decay of $\pi$-mesons of the energy $\varepsilon$, muons are generated in some energy interval from $E_{\min}$ to $E_{\max}$.

Multiplying the number of decaying $\pi$-mesons of energy $\varepsilon$, by the probability of generation

$$W(\varepsilon, E)dE\sim\frac{dE}{\varepsilon\sqrt{1-\frac{m_\pi^2}{\varepsilon^2}}},$$

We find the number of muons with the energy $E$, produced by the decay of $\pi$-mesons with the energy $\varepsilon$ at the height $x$.

$$\mu(E, \varepsilon, x)d\varepsilon dE dx=B_0 e^{-\beta x}\frac{x, \varepsilon^{-(3+\gamma_\pi)}}{\left(1+\frac{E_\pi}{\varepsilon}\right)\sqrt{1-\frac{m_\pi^2}{\varepsilon^2}}}\left[1-\frac{ax}{2+\frac{E_\pi}{\varepsilon}}+\frac{a^2x^2}{\left(2+\frac{E_\pi}{\varepsilon}\right)\left(3+\frac{E_\pi}{\varepsilon}\right)}+\cdots\right]d\varepsilon dE dx. \tag{6}$$

Muons of the energy $E$ are produced by $\pi$-mesons of the energy $\varepsilon$, $\varepsilon_-\leq\varepsilon\leq\varepsilon_+$ where $\varepsilon_+=m_\pi(EE^*+PP^*)/m_\mu^2$ is the maximum energy of *a* $\pi$-meson generating a muon of energy $E$, $\varepsilon_-=m_\pi(EE^*+PP^*)/m_\mu^2$, the minimum energy of *a* $\pi$-meson generating a muon of energy $E$.

The average value of polarization of $\mu$-meson of the energy $E$, generated at the depth $x$,

$$\eta(E, x)=\frac{EE^*}{PP^*}-\frac{m^2\beta}{m_\pi PP^*}$$

$$\times\frac{\displaystyle\int_{\varepsilon_-}^{\varepsilon_+}\frac{\varepsilon^{-(2+\gamma_\pi)}}{\left(1+\frac{E_\pi}{\varepsilon}\right)\sqrt{1-\frac{m_\pi^2}{\varepsilon^2}}}\left(1-\frac{ax}{2+\frac{E_\pi}{\varepsilon}}+\frac{a^2x^2}{\left(2+\frac{E_\pi}{\varepsilon}\right)\left(3+\frac{E_\pi}{\varepsilon}\right)}\cdots\right)d\varepsilon}{\displaystyle\int_{\varepsilon_-}^{\varepsilon_+}\frac{\varepsilon^{-(3+\gamma_\pi)}}{\left(1+\frac{E_\pi}{\varepsilon}\right)\sqrt{1-\frac{m_\pi^2}{\varepsilon^2}}}\left(1-\frac{ax}{2+\frac{E_\pi}{\varepsilon}}+\frac{a^2x^2}{\left(2+\frac{E_\pi}{\varepsilon}\right)\left(3+\frac{E_\pi}{\varepsilon}\right)}\cdots\right)d\varepsilon}. \tag{7}$$

The first approximation $\kappa(\varepsilon)=\beta$ corresponds to the physical assumption that nuclear interactions and regeneration of $\pi$-mesons are of a character *t* at leads their absorption by the atmosphere according to the some exponential law as that for absorption of nucleons.

In this case diffusion eq. (2) allows the solution in the form of a finite the algebraic function, derived from (5) if we take $a=0$, then

$$\eta(E)=\frac{EE^*}{PP^*}-\frac{m_\beta^2}{m_\pi PP^*}\cdot\frac{\displaystyle\int_{\varepsilon_-}^{\varepsilon_+}\frac{\varepsilon^{-(2+\gamma_\pi)}}{\left(1+\frac{E_\pi}{\varepsilon}\right)\sqrt{1-\frac{m_\pi^2}{\varepsilon^2}}}d\varepsilon}{\displaystyle\int_{\varepsilon_+}^{\varepsilon_-}\frac{\varepsilon^{-(3+\gamma_\pi)}}{\left(1+\frac{E_\pi}{\varepsilon}\right)\sqrt{1-\frac{m_\pi^2}{\varepsilon^2}}}d\varepsilon} \tag{8}$$

Thus, in the first approximation the polarization of muons is independent of their production height. It can be explained by the fact that eq. (2) in this approximation allows the solution in the separate variables $\varepsilon$ and $x$, *i.e.* the energy spectrum of $\pi$-mesons is equal at all heights. The dependence of the polarization of the muon produced on their energy is given in the plot (curve I) on the semilogrithmical scale.

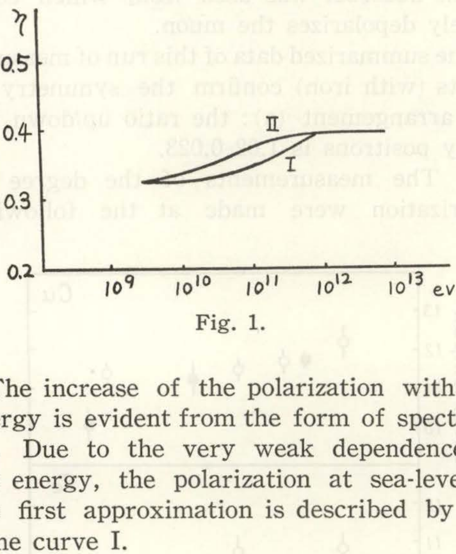


Fig. 1.

The increase of the polarization with the energy is evident from the form of spectrum (5). Due to the very weak dependence on the energy, the polarization at sea-level in the first approximation is described by the same curve I.

**On the precise value of $\mu$-meson polarization at sea level**

We calculated the muon polarization of the energy $E$ at different depths of atmosphere $x$ and with different assumptions about the character of absorption of $\pi$-mesons (the value of $x$ was varied from $\beta=0.65$ to 1). The calculations were made according to formula (7) and the sign-alternating and rapidly decreasing now allowed the necessary accuracy to be obtained.

The calculations showed, that the polarization increase with the depth $x$ of atmosphere and with the absorption coefficient $\kappa$. The physical sense of it is not difficult to understand.

The polarization increase with the sharpness of the $\pi$-meson energy spectrum. The spectrum of $\pi$-mesons at the given height is determined by the $\pi$-meson generation spectrum and by the spectrum of $\pi$ mesons, coming from higher depth. But the latter is less sharper, due to the decay of $\pi$-mesons. Furthermore less is the contribution of $\pi$-mesons arriving from higher depths, and consequently, the denser the atmosphere the greater the absorption of $\pi$-mesons, the sharper is the resulting spectrum. The first approximation, we received referd to the case, when the polarization has the least value as $ax=0$. Another deliberately overestimated value for polarization is shown in the plot curve I here we put $a=0.35$, $1.9_{n.u.}\leq x, \leq 2.8_{n.u.}$. The polarization of muons measured at sea level is less than estimated from curve II, as $a(\varepsilon)\leq 0.35$ and the effective depth of $\pi$ meson production $x<x_1$, for the energies under consideration. Hence, the polarization of $\mu$ mesons of the energy $E\gg 2.10^9$ ev, at sea level is described by the curve lying between the curve I and II.

With varying the experimentally measured parameters $\Delta\gamma_n=\pm 0.05$ and $\Delta E_n=\pm 3.10^{10}$ ev the polarization does considerably not change $\Delta\eta - 0.01$ and the sign of quality corresponds to the most unfavorable combination of errors.

In conclusion I would like to avail myself of the opportunity to express my gratitude to Prof. G. T. Zatsepin for his supervision of my work and to V. T. Ritus for most useful discussion.