JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN Vol. 17, SUPPLEMENT A-III, 1962
INTERNATIONAL CONFERENCE ON COSMIC RAYS AND THE EARTH STORM Part III

III-5-16. On Hard Showers Produced by μ-mesons

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It is shown that the total cross section of hard showers produced by a μ-meson or an electron is limited up to two unknown functions by using Lorentz and Gauge invariances only. The character of the strong interactions is confined in these functions. From the viewpoint of this fact, some preliminary analysis have been done in order to investigate what property the strong interaction should have to explain the results of the Osaka experiments, especially the spectrum of the transferred energy from the μ-meson and of square of the transferred four momentum.

1. Measurements on multiple π-meson production by underground μ-mesons have recently been made by members of Osaka City University[1]. One of their data is on the integral spectrum of the transferred energy from the μ-meson to the target nucleon, ε. Another data is on the integral spectrum of square of transferred *4*-dimensional momentum, q^2, namely $q^2=|\boldsymbol{q}|^2-\varepsilon^2$ where $\boldsymbol{q}$ is the tarnsferred momentum. These spectra depend on the strong interaction between the target nucleon and the produced particles as well as on the μ-meson current. Our knowledge about the strong interaction is insufficient to derive these spectra theoretically. Theoretical analyses so far performed[2),3),4)] are all based on special assumptions about the strong interaction*, hence are inadequate for the standpoint of analyses of these spectra. Therefore we will set up the problem as follows: If μ-mesons interact with electromagnetic field as electrons do, what property should the strong interaction have to explain the obtained experimental results? From such a viewpoint, we derive a general expression of the cross section concerned in the next section. Using this expression we make some preliminary analyses of the present experiment in § 3 and § 4. The last section contains some concluding remarks.

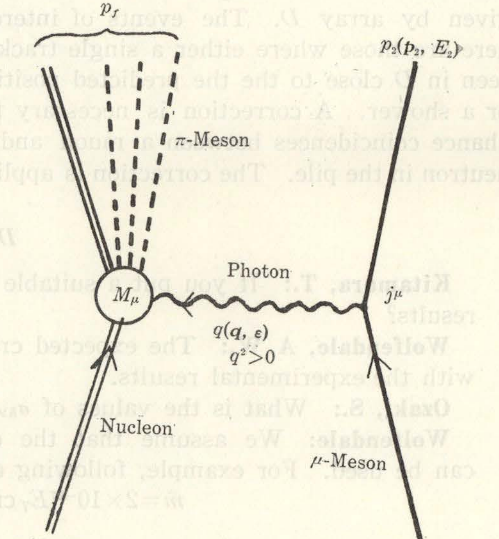

Fig. 1. The Feynman diagram for an inelastic collision between a μ-meson and a nucleon.

2. The process of what we concern is represented by the Feynman diagram in Fig. 1. An incident μ-meson (electron) with momentum $p_1(\boldsymbol{p}_1, E_1)$ collides with the rest nucleon $p(\boldsymbol{o}, M)$ and emits a virtual photon $q(\boldsymbol{q}, \varepsilon)$ and has momentum $p_2(\boldsymbol{p}_2, E_2)$ in the final state. The total cross section of this process is given as

$$\sigma=(2\pi)^2(E_1/|\boldsymbol{p}_1|)\int d\boldsymbol{p}_2(d^4q/q^4)L_{\mu\nu}J^{\mu\nu}, \quad (1)$$

* Details of this point were reported at the plenary session on μ-meson, as a comment.

where

$$J^{\mu\nu}=\sum_{(2)}\overline{\sum_{(1)}}j^{\mu*}j^{\nu}\delta(p_2+q-p_1)\,, \qquad (2)$$

and

$$L_{\mu\nu}=\sum_{f}\overline{\sum_{i}}M_\mu{}^*M_\nu\delta(p_f-p-q)\,. \qquad (3)$$

Here M_μ and j^μ represent nucleon vertex and μ-meson current and p_f is the total momentum of the final nucleon and the produced π-mesons. $\overline{\sum_{(1)}}$ and $\overline{\sum_{i}}$ mean averaging on spins of the incident μ-meson and the initial nucleon, respectively, $\sum_{(2)}$ means summing up the final spin of the μ-meson, and $\sum_f$ represents summing up the spin and also the integration by the momenta of the final nucleon and produced π-mesons.

$L_{\mu\nu}$ must have Lorentz covariant form such as

$$L_{\mu\nu}=L^{\mathrm{I}}p_\mu p_\nu+L^{\mathrm{II}}p_\mu q_\nu$$
$$+L^{\mathrm{III}}q_\mu p_\nu+L^{\mathrm{IV}}q_\mu q_\nu+L^{\mathrm{V}}g_{\mu\nu}\,, \qquad (4)$$

where $g_{\mu\nu}$ is the metric tensor and L's are functions of Lorentz invariant quantities $q^2, pq(=-M\varepsilon)$ and $p^2(=-M^2)$. By virtue of the symmetry property $(L_{\mu\nu}{}^*=L_{\nu\mu})$ and Gauge invariance $(L_{\mu\nu}q^\nu=0)$, only two functions among the above five are independent and $L_{\mu\nu}$ reduces to

$$L_{\mu\nu}=(L/M^2)\{q^2p_\mu p_\nu$$
$$-(pq)(p_\mu q_\nu+q_\mu p_\nu)+(pq)^2g_{\mu\nu}\}$$
$$+L'(q_\mu q_\nu-q^2g_{\mu\nu})\,. \qquad (5)$$

Here the terms proportional to q_μ and q_ν do not contribute to the cross section because of the conservation of the μ-meson current: $q_\mu j^\mu=0$. The character of the strong interaction is confined in these two Lorentz invariant functions L and L'. $J^{\mu\nu}$ is calculated as usual.

After some calculation the differential cross section as to ε and q^2 is given by

$$\frac{d^2\sigma}{dq^2d\varepsilon}=\frac{\alpha}{8\pi^2}\frac{1}{|\boldsymbol{p}_1|^2}\frac{1}{q^4}$$
$$\times\Bigg[L\left\{(E_1{}^2+E_2{}^2)q^2-2m^2\varepsilon^2-\frac{1}{2}q^4\right\}$$
$$+L'(2m^2-q^2)q^2\Bigg]\,, \qquad (6)$$

where α is the fine structure constant and m the mass of μ-meson (electron). This expression is the starting point of our analysis.

3. Following Williams-Weizsäcker's[2] (W-W) idea, we can obtain some information about L and L' from the corresponding process by a real photon (Fig. 2). The cross section of this process, $\sigma_{h\nu}$, is given by

$$\sigma_{h\nu}=(1/4\pi\varepsilon)\sum_f\overline{\sum_i}\overline{\sum_e}|e_\mu M^\mu|^2\delta(p_f-p-q)|_{q^2=0}$$
$$=(1/4\pi\varepsilon)[L\varepsilon^2-L'q^2]_{q^2=0}\,, \qquad (7)$$

where e_μ is the polarization vector of the incident photon and $\overline{\sum_e}$ means averaging on it. If L' is not singular at $q^2=0$, it does not contribute to $\sigma_{h\nu}$.

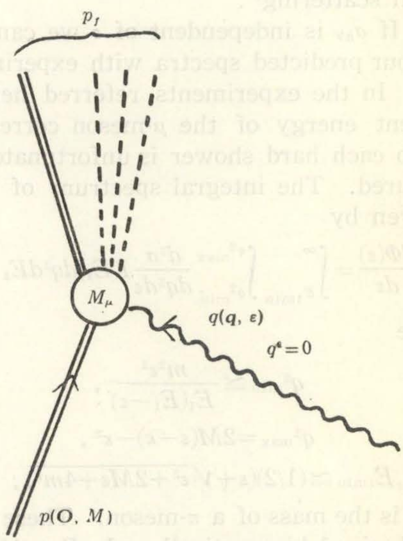

Fig. 2. The Feynman diagram for a photo-meson production.

Even if we assume the constancy of $\sigma_{h\nu}$ at high energy which is an assumption usually adopted, our knowledge about L and L' would not be enough to deduce the spectra, $d\sigma/d\varepsilon$ and $d\sigma/dq^2$, etc., from Eq. (6). Therefore we must induce L and L' from the experiment now concerned. In other words, we must seek the character which the strong interaction should have in order to explain the obtained experimental results.

From such a point of view, we examine, in order to find the first clue, the simple case* that L' equals zero and hence

$$L_{q^2=0}\equiv L_0=(4\pi/\varepsilon)\sigma_{h\nu}\,.$$

* In the other extreme case that $L=0$ and $L'q^2|_{q^2=0}=4\pi\varepsilon\,\sigma_{h\nu}$, we obtained a result quite different not only from W-W's picture but also from the experimental results.

Further we choose

(a) $$L(\varepsilon, q^2)=L_0=(4\pi/\varepsilon)\sigma_{h\nu}\,, \tag{8}$$

or

(a) $$L(\varepsilon, q^2)=L_0\left(\frac{\Lambda^2}{q^2+\Lambda^2}\right)^2$$

$$=\frac{4\pi}{\varepsilon}\sigma_{h\nu}\left(\frac{\Lambda^2}{q^2+\Lambda^2}\right)^2, \tag{9}$$

$$\Lambda^2=0.365 \quad (\mathrm{Bev})^2,$$

where the value of Λ^2 is taken only as the first clue which corresponds to the radius of proton charge distribution, *i.e.* 0.8×10^{-13} cm, obtained by the experiments on electron-proton scattering[5].

4. If $\sigma_{h\nu}$ is independent of ε we can compare our predicted spectra with experimental data. In the experiments referred here the incident energy of the μ-meson corresponding to each hard shower is unfortunately not measured. The integral spectrum of ε, $\Phi(\varepsilon)$ is given by

$$\frac{d\Phi(\varepsilon)}{d\varepsilon}=\int_{E_{1\min}}^{\infty}\int_{q^2_{\min}}^{q^2_{\max}}\frac{d^2\sigma}{dq^2d\varepsilon}I(E_1)dq^2dE_1\,, \tag{10}$$

where

$$q^2_{\min}\cong\frac{m^2\varepsilon^2}{E_1(E_1-\varepsilon)}\,,$$

$$q^2_{\max}=2M(\varepsilon-\kappa)-\kappa^2\,,$$

$$E_{1\min}\cong(1/2)(\varepsilon+\sqrt{\varepsilon^2+2M\varepsilon+4m^2})\,,$$

and κ is the mass of a π-meson. These limits are obtained kinematically. In Eq. (10) $I(E_1)$ stands for the differential spectrum of the incident μ-mesons and is experimentally given in the following approximate form,

$$I(E)=0.392/(E+15)^3$$

$$(\mathrm{cm}^2\cdot\mathrm{sec}\cdot\mathrm{sterad}\cdot\mathrm{Bev})^{-1},\ E:\ \mathrm{Bev\ unit.} \tag{11}$$

We get the integral spectrum of q^2, $\Psi(q^2)$, in the same way, *i.e.*

$$\frac{d\Psi(q^2)}{dq^2}=\int_{\varepsilon_{\min}}^{\infty}\int_{E_{1\min}}^{\infty}\frac{d^2\sigma}{dq^2d\varepsilon}I(E_1)dE_1d\varepsilon\,, \tag{12}$$

where

$$E_{1\min}=(1+\sqrt{1+4m^2/q^2})(\varepsilon/2)$$

and

$$\varepsilon_{\min}=\kappa+(q^2+\kappa^2)/2M\,.$$

Fig. 3 shows the integral spectrum of ε, $\Phi(\varepsilon)$. The dashed line, representing the case (b) (Eq. (9)), is drawn only on rough estimate. The experimental histogram does not seem to determine which is best among the four curves. Here $\sigma_{h\nu}$ takes the values 2.6~2,

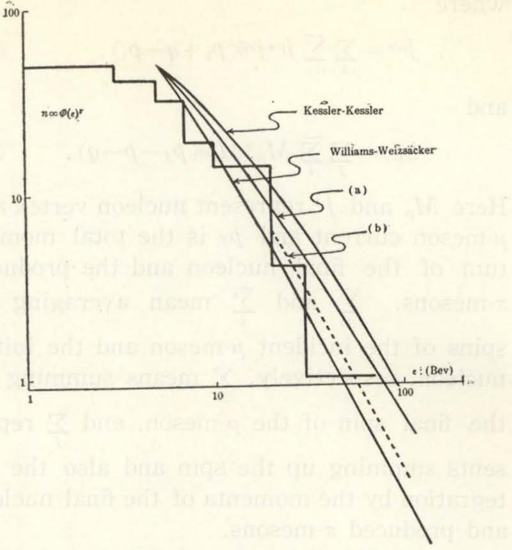

Fig. 3. Integral spectrum of the transferred energy, ε from μ-mesons.

0.62 and 0.24×10^{-28} cm² in W-W, (b), (a) and Kessler-Kessler[4] (K-K), respectively, for normalization.

The integral spectrum of square of the transferred 4-dimensional momenta $\Psi(q^2)$ is shown in Fig. 4.* The original W-W method cannot give this spectrum, while the K-K method gives almost the same curve as (a). The curves are normalized by taking for $\sigma_{h\nu}$ about one third of the corresponding values in the above. Since the experiments referred here omit the events whose transferred energies are lower than about 5 Bev, the theoretical curves should be depressed at smaller values of q^2 than several (Bev)² and then $\sigma_{h\nu}$ may be same as the above values. Taking this circumstance into account we can see from Fig. 4 that (b) is in disagreement and (a) is in good agreement with the experimental histogram.

5. Although L in (b) has a similar cutoff factor to that obtained by low energy electron-proton scattering, it is premature to conclude that the character of the strong interaction in the present process are very

* The q^2 of each event is a function of E_1, ε, θ (the deflection angle of the incident μ-meson). ε and θ were measured but E_1. Therefore the experimetal histogram is drawn on the expectation values of q^2, *i.e.* $\bar{q}^2=\int q^2(E_1,\varepsilon,\theta)\,I(E_1)\,dE_1\Big/\int I(E_1)\,dE_1$.

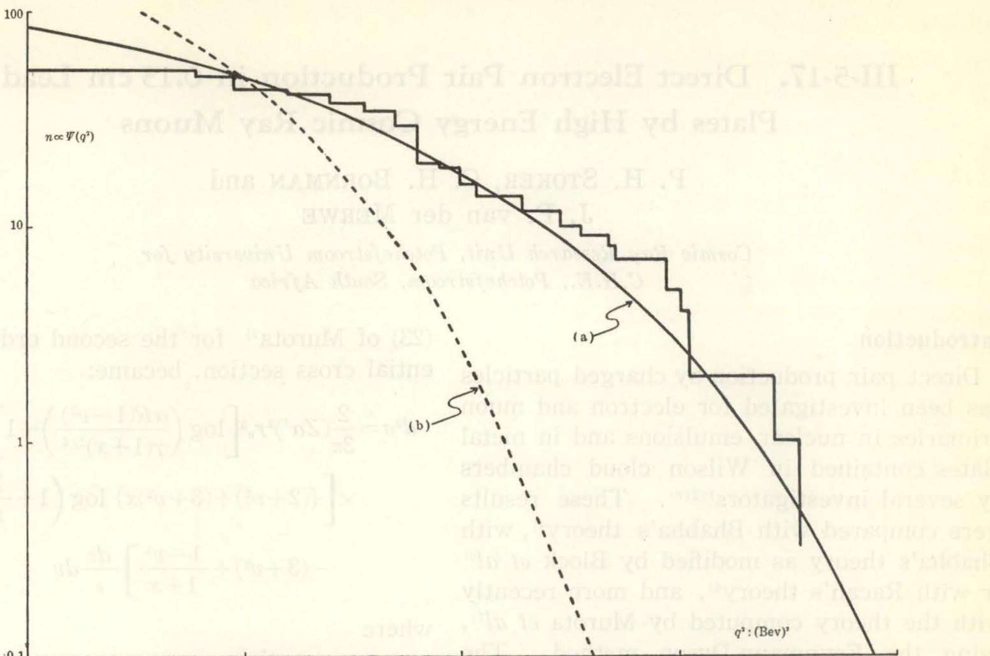

Fig. 4. Integral spectrum of the square of the transferred 4-dimensional momentum.

different from that in the electron-proton scattering because of differences between inelastic and elastic scatterings and difference of magnitude of q^2.

Our analysis is quite preliminary and there may still be the possibility to induce some more information from the present experimental results. If many kinds of experimental data are further accumulated we shall know the more detailed character of L and L', hence the structure of the strong interaction. The inelastic electron-nucleon collisions in high energy region may throw light on whether a μ-meson has some strange properties or not.

References

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4) D. Kessler and P. Kessler: Nuovo Cimento **6** (1956) 601. P. Kessler: *ibid* **17** (1960) 809.
5) For example, R. Hofstadter and R. R. Wilson: Proceedings of the Tenth Annual International Conference on High-Energy Physics at Rochester (1960).

Discussion

Menon, M. G. K.: What physical meanings are for L and L'?

Messel, H.: Please, discuss after the session.