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Theory of Magnetic Relaxation

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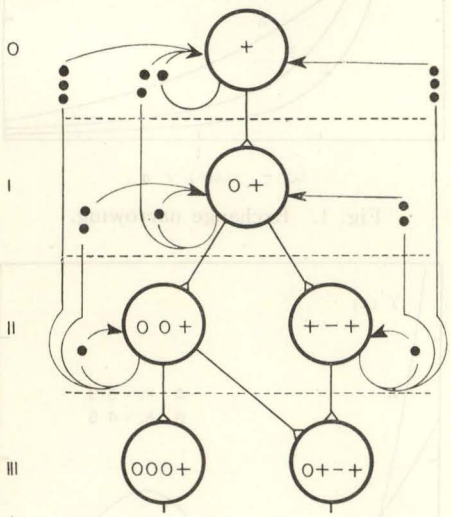
It is proposed to use the method of response function, or the two-time Green's function, for a systematic study of magnetic relaxation phenomena including the anomaly around the critical point.

In analogy with the relation between moments and cumulants in the theory of probability ordinary Green's functions[1] are reduced to those of the cumulant type in every order. Then at an appropriate order a stochastic assumption is introduced and the higher order Green's function is replaced by a function of Gaussian type, conforming to the initial time behaviour of the exact function. This method is wider in applicability than the ordinary perturbational approach to the extent that the damping phenomena which are not of Boltzmann-Bloch type may be included.



Hierarchy of Green's function.

The case of Heisenberg magnet is treated as an example, and the following Hamiltonian is adopted.

$$H=-\sum_n \omega_n S_n{}^0+\sum_{n>m}\sum J(n-m)(\boldsymbol{S}_n\cdot\boldsymbol{S}_m)$$
$$+\sum_{n>m}\sum_{\mu}\sum F^{\mu}(n-m)\{nm\}_{\mu}-K\sum_n (S_n{}^0)^2$$

where the individual term stands for Zeeman, exchange, dipolar and anisotropy energies, respectively.

I. The case of single sublattice.

The simplest necessary approximation is of the third order, and the Green's function $\langle[S^+(x_n,\ t),\ S^-(x_m,\ t)]\rangle$ yields the imaginary part of the susceptibility

$$\chi''_{+-}(k,\omega)\propto\frac{\alpha(k,\omega)\Gamma_2(k,\omega)+\beta_2(k,\omega)\{\omega-\Delta(k,\omega)\}}{\{\omega-\Delta(k,\omega)\}^2+\{\Gamma_2(k,\omega)\}^2}$$

where

$$\Delta(k,\omega)=\Delta_1(k)+\Delta_2(k,\omega),$$
$$\alpha(k,\omega)=\alpha_1(k)+\alpha_2(k,\omega),$$

and

$$\Delta_1(k)=\omega_0(0)+\frac{\sigma(0)}{N}(2K+A(k)+B(0)),$$

$$\Delta_2(k,\omega)=\frac{1}{N}\sum_q\{a^2(k,q)F_s(e^{-(b^2(k,q)t^2)/2})$$
$$-c^3(k,q)F_c(te^{-(b^2(k,q)t^2)/2})\},$$

$$\Gamma_2(k,\omega)=\frac{1}{N}\sum_q\{a^2(k,q)F_c(e^{-(b^2(k,q)t^2)/2})$$
$$+c^3(k,q)F_s(te^{-(b^2(k,q)t^2)/2})\}.$$

α_1, α_2 and β_2 have expressions similar to Δ_1, Δ_2 and Γ_2 respectively. For $k=0$ the above expression describes the resonance absorption. The damping constant $\Gamma_2(0,\omega)$ stands for the line width, which is now ω dependent. For $k\neq 0$ the constant of phase-diffusion[2] may be defined by

$$D_+=\left(\frac{\partial^2\Gamma_2}{\partial k^2}\right)_{k=0,\ \omega=0}$$

In an analogous manner the constant D_0 of polarization-diffusion, i. e., the ordinary spin-diffusion,[3] may be obtained from the Green's function $\langle[S^0(x_n,t),\ S^0(x_m,\ t')]\rangle$.

The quantities a^2 and b^2 are expressed in terms of the spatial correlation of the spin deviations as follows.

$$a^2{}_{(0)}=2K^2\gamma^{00}_{(0)}+\tfrac{3}{2}\sum_n\sum_m F(n)F(m)$$
$$\times\{\gamma^{00}_{(n,m)}+\tfrac{1}{4}\gamma^{-+}_{(n,m)}\},$$

$$b^2{}_{(0)}=2K^2\gamma^{00}{}_{(0)}+\sum_n\sum_m J(n)J(m)$$
$$\times\{\gamma^{00}{}_{(n,m)}+\tfrac{1}{2}\gamma^{-+}{}_{(n,m)}+\gamma^{+-}{}_{(n,m)}$$
$$+\delta_{(n,m)}(\gamma^{00}{}_{(n)}+\tfrac{1}{2}\gamma^{-+}{}_{(n)}+\gamma^{+-}{}_{(n)})\}$$
$$+\tfrac{1}{8}\sum_n\sum_m F(n)F(m)\{8\gamma^{00}{}_{(n,m)}+\gamma^{-+}{}_{(n,m)}-2\gamma^{+-}{}_{(n,m)}$$
$$+\delta_{(n,m)}(4\gamma^{00}{}_{(n)}-2\gamma^{-+}{}_{(n)})-2\gamma^{+-}{}_{(n)}\}\,.$$

Here the correlation of spin deviations[4] are defined by

$$\gamma^{\alpha\beta}{}_{(x)}=\langle S^\alpha(x)S^\beta(0)\rangle-\langle S^\alpha(x)\rangle\langle S^\beta(0)\rangle$$
$$=\frac{1}{N}\sum_q e^{iqx}\gamma^{\alpha\beta}_{q,-q}$$
$$\gamma^{\alpha\beta}{}_{(x,\,y)}=\gamma^{\alpha\beta}{}_{(x-y)}+\gamma^{\alpha\beta}_{(x+y)}$$

and in the first order approximation it is found that

$$\gamma^{+-}_{q,-q}=\frac{2\sigma(0)}{1-e^{-\beta J_1(q)}}\,,\quad \gamma^{-+}_{q,-q}=\frac{2\sigma(0)}{e^{\beta J_1(q)}-1}\,,$$
$$\gamma^{00}_{q,-q}\simeq\frac{2\sigma(0)}{e^{\beta J_1(q)}-e^{-\beta J_1(q)}}\,.$$

The situation above the Curie point may be described in the long wave approximation. $\gamma^{\pm\mp}$ and γ^{00} is then given by

$$\gamma^{\pm\mp}{}_{(x)}=2\gamma^{00}{}_{(x)}=\frac{N}{2}(\kappa_0{}^2+\tfrac{1}{2}q_M{}^2)\frac{3}{q_M{}^3}$$
$$\times\frac{1}{x}\int_0^{q_M}dq\,\frac{q\sin qx}{\kappa_0{}^2+q^2}\propto\frac{e^{-\kappa_0 x}}{x}$$
$$\gamma^{\pm\mp}{}_{(0)}=2\gamma^{00}{}_{(0)}=\frac{N}{2}(\kappa_0{}^2+\tfrac{1}{2}q_M{}^2)\frac{3}{q_M{}^3}$$
$$\times\{q_M-\kappa_0\tan^{-1}(q_M/\kappa_0)\}\,,$$

where

$$\kappa_0{}^2=\frac{N\omega_0(0)}{J\zeta\,d^2\sigma(0)}=\frac{N\mu^2}{J\zeta d^2\chi(0)}=\lambda_0{}^{-2}$$

In the same approximation the sum rule yields the temperature dependence of κ_0, i.e., the inverse of the correlation length λ_0.

In the high temperature limit the correlation is strictly localized, i.e., $\kappa_0\to\infty$. Ordinary Gaussian type absorption is obtained for the case of weak exchange[5], whereas Lorentzian shape is found in the case of strong exchange[6] (exchange narrowing).

When we approach the Curie point from the above the spatial correlation begins to have a tail owing to the appearance of the short range order, and the damping increases generally. It is found that κ_0 tends to nought, namely the correlation length be-

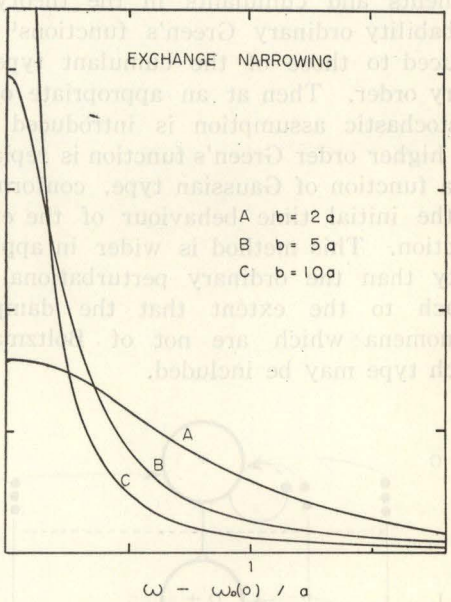
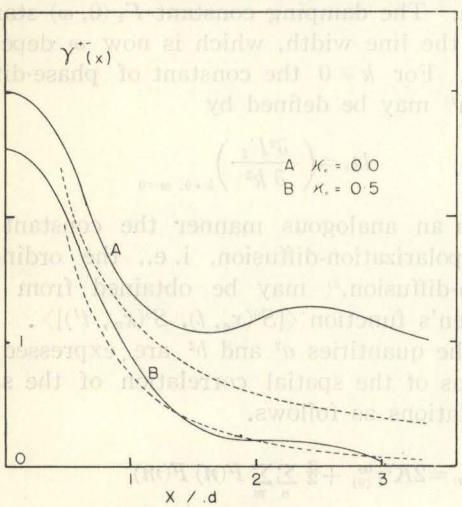
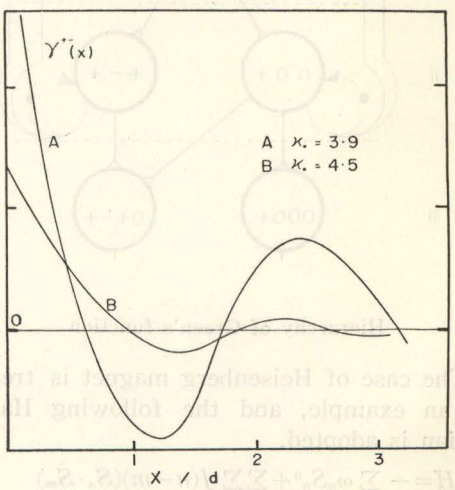


Fig. 1. Exchange narrowing.

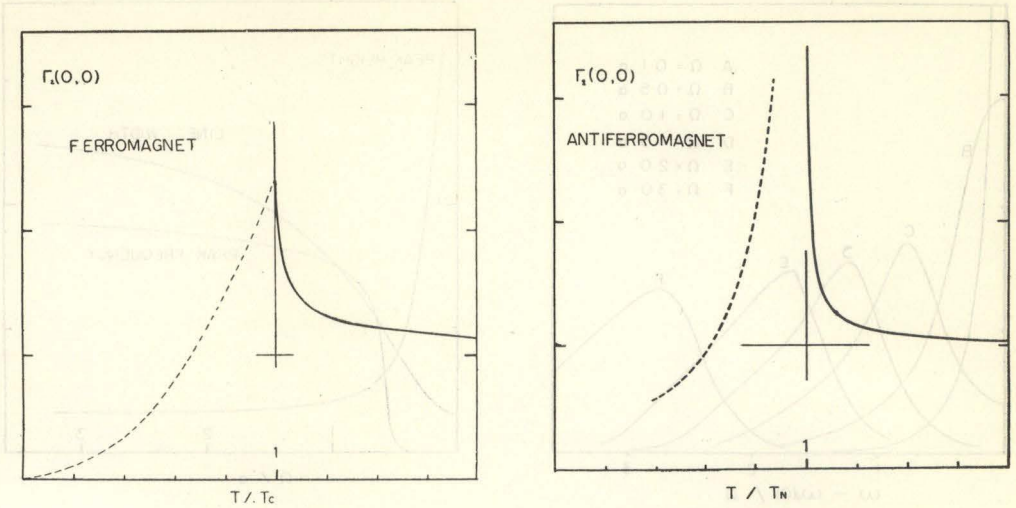


(a) Ferromagnet.



(b) Antiferromagnet.

Fig. 2. Spatial correlation.



(a) Ferromagnet. (b) Antiferromagnet.

Fig. 3. Dependence of $\Gamma_2(0, 0)$ on temperture.

comes arbitrarily large. This fact is reflected in the behaviour of the line width, which now increases in the neighbourhood of Curie point and reach a value roughly 1.7 times as large as that in the high temperature limit[7], provided we retain adiabatic perturbations only. It is to be noted, however, that there is no divergence at the Curie point even if there is exchange interaction only.

When, on the other hand, we get away from Curie point towards the lower temperature side, the spatial correlation begins to be localized and the damping decreases again. This is due to the fact that the damping is characterized by the correlation of the deviation from the long range order. It is natural, therefore, to find a negligible damping at very low temperature according to our formulae

$$\gamma^{-+}_{q,-q}=\gamma^{00}_{q,-q}\simeq 2\sigma(0)\, e^{-\beta\Delta_1(q)}$$

These results are consistent with the spin wave theory[8].

The quantitative analysis of the intermediate temperature range is still in progress.

II. The case of two sublattices

The only complication in this case lies in the fact that we have to treat the alternant mode as well as the uniform mode as being coupled to each other.

In the paramagnetic range the qualitative behaviour of an antiferromagnet is similar to that of a ferromagnet, we get exactly the same expression for χ''_{+-} as in the case of single sublattice. Thus in the highest temperature limit we get the same exchange-narrowed width as before. The actual value of the correlation of deviation, however, is not quite the same as in the previous case, because of the difference in sign of the exchange interaction. In the long wave approximation they are given by

$$\gamma^{\pm\mp}{}_{(x)}=2\gamma^{00}{}_{(x)}=\frac{N}{2}(\kappa_0{}^2-\tfrac{1}{2}q_M{}^2)\frac{3}{q_M{}^3}\times\frac{1}{x}\int_0^{q_M}dq\frac{q\sin qx}{\kappa_0{}^2-q^2}$$

$$\gamma^{\pm\mp}{}_{(0)}=2\gamma^{00}{}_{(0)}=\frac{N}{2}(\kappa_0{}^2-\tfrac{1}{2}q_M{}^2)\times\frac{3}{q_M{}^3}\left\{\frac{\kappa_0}{2}\ln\frac{|\kappa_0+q_M|}{|\kappa_0-q_M|}-q_M\right\}$$

where κ_0 is defined by the same formula as above. The temperature dependence of κ_0 is again obtained from the sum rule. In the present case, however, κ_0 tends to $q_M(\sim\pi/d)$, namely the correlation length becomes of the order of lattice constant, when we approach to the Néel point. As a result the damping increases around Néel point[9]. The main difference from the ferromagnetic case lies in the fact that the range of anomalous increase is narrower and that there is a possibility of divergence in the damping at the Néel point, although the introduction of anisotropy will remove it mathematically. The quantitative analysis of the behaviour below

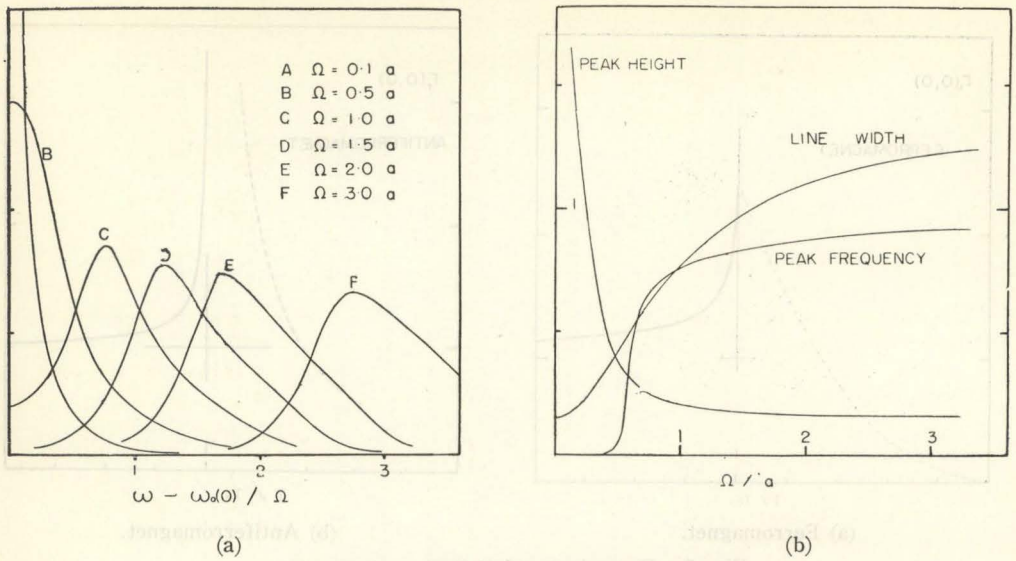


Fig. 4. Exchange amalgamation.

the Néel point is in progress.

Another example of the same type treatment is the case of a paramagnet with alternant g-factors[10]. The amalgamation phenomenon due to the exchange interaction existing between the two species is treated in the same framework. The change in line shape and width is also derived from the theory. The result is shown in the figure.

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DISCUSSION

C. KITTEL: An extension of a result of de Gennes suggests that near Curie point the line width in ferromagnetic resonance is substantially smaller than the exchange-narrowed value, but increases monotonically with temperature to approach as $T \to \infty$ the exchange-narrowed value. Experiments on Ni, Fe and YIG appear to agree qualitatively with this picture.

K. TOMITA: We are aware of the data you mentioned. But can Ni and Fe which are metals be described by Heisenberg localized model safely? With this question in mind, I did not claim strongly that we are ready to explain any real example in the case of ferromagnet. I just gave the theoretical results for ferromagnetic coupling which stands in an interesting contrast with the case of antiferromagnetic coupling. Up to the present we have only included the adiabatic part of the dipolar energy, so that the inclusion of the non-adiabatic part might change the situation.

R. J. ELLIOTT: I had also on a much ruder calculation thought that the ferromagnetic resonance line width would peak at T_c. However the experimental results indicate that the line width increases sharply above T_c, and an extension by Prof. Kittel of an earlier theory does give this result. It would be very interesting to resolve these differences.