JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN VOL. 17, SUPPLEMENT B-I, 1962
PROCEEDINGS OF INTERNATIONAL CONFERENCE ON MAGNETISM AND CRYSTALLOGRAPHY, 1961, VOL. I

The Skin-Effect in Iron Sheets

J. E. L. BISHOP, E. W. LEE AND
L. ALBERTS*

University of Sheffield, England

Measurements of the resistance and inductance of iron strips whose thickness ranges from 7 to 93 microns have been made over the frequency range 0 to 30 Mc/s. Results are interpreted in terms of a domain model identical with that used by Polivanov to calculate the complex permeability of ferromagnetic sheets. Agreement between experiment and simple theory is only moderate. Quite good agreement may be obtained, however, with the theory extended to take into account the fact that different domain walls will, in general, have different restoring forces.

1. Introduction

Although the initial permeability of ferromagnetic materials is easily measurable at low audio and at microwave frequencies, measurement in the intermediate frequency range usually presents some difficulty. Attempts have been made to circumvent these difficulties by measuring the skin-effect whereby the resistance and the inductance of a wire or strip is determined as a function of frequency. In general these measurements are easier to carry out and frequently may be made over an extensive range of frequencies with a single measuring instrument. However, there remains the problem of relating the measured resistance or inductance to the initial permeability. This is usually done by choosing a sample of standard geometrical shape, e. g., a long wire, for which calculations based on the appropriate solutions of Maxwell's equations have been made, for example by Kelvin. Unfortunately these calculations all assume the permeability to be uniform and there now exists overwhelming evidence to show that this assumption seriously underestimates eddy-current effects, particularly when the smallest physical dimension of the sample becomes comparable with the average distance between domain walls. Thus it is necessary to treat the skin-effect as a problem in its own right.

2. Theory

We have calculated the resistance and inductance of an infinite sheet as a function of frequency using as a simple model the case in which the sheet is divided by 180° walls into domains magnetized at right angles to the direction of the current flow. The domain walls must be deformed by the field produced by the current but in such a way that the resultant magnetization is zero. The calculations, based on the assumption (Polivanov, 1952) that the intra-domain permeability is zero, are very similar to those previously given by various authors for the frequency dependence of the initial permeability.

The effects of spin damping and of domain wall stiffness have been investigated. In order to incorporate the latter it is necessary to make some assumption concerning the shape of a moving domain wall. One of the simplest, and one which seems physically plausible is that the domain walls are firmly held at the surface by imperfections. Making this assumption one finds that the fractional increase in resistance due to the eddy currents is

$$U=\frac{R_\omega-R_0}{R_0}=\frac{2d}{\pi a}\sum_{even}\frac{1}{n}\cdot\frac{\coth\frac{n\pi a}{d}+\frac{n\beta}{32\pi M_s^2\sigma d}}{\left(\coth\frac{n\pi a}{d}+\frac{n\beta}{32\pi M_s^2\sigma d}\right)^2+\left(\frac{n[n^2\pi^2\gamma^2+\alpha d^2]}{32\pi M_s^2 d^3\sigma\omega}\right)^2}$$

and

$$L=\frac{1}{16\pi^2\omega^2M_s^2ad^3\sigma^2}\sum_{even}\frac{n^2\pi^2\gamma+ad^2}{\left(\coth\frac{n\pi a}{d}+\frac{n\beta}{32\pi M_s^2\sigma d}\right)^2+\left(\frac{n[n^2\pi^2\gamma+\alpha d^2]}{32\pi M_s^2 d^3\sigma\omega}\right)^2}$$

* On leave from the University of the Orange Free State, S. A.

is the inner inductance of unit length and width of the sheet. Here d is the thickness of the sheet, $2a$ is the domain width, M_s is the spontaneous magnetization, α is the restoring force constant per unit area of Bloch wall, β is the spin damping constant, γ is the Bloch wall energy per unit area, σ is the electrical conductivity and the summation is over all even values of n.

3. Experiment

In obtaining experimental data for comparison with the theory one must be careful not to overlook the fact that the high frequency resistance of a sheet of finite width *never* approximates to that of an infinite sheet even when the ratio width to thickness tends to infinity. This is due to the concentration of current at the edges and is exactly analogous to the concentration of charge at the edges of an electrified conductor. We sought to overcome this by folding the sample back on itself an even number of times thereby making the current density more uniform (Bishop, 1962). The material chosen was pure iron, carefully annealed in dry hydrogen, because it was felt that, with a high anisotropy and therefore low-intra-domain permeability and high wall mobility this comes closest to the conditions assumed in the calculations. Measurements were made on strips of various thicknesses between 7μ and 93μ over the frequency range 0 to 30 Mc/s. Measurements of the complex permeability at 2 Kc/s. were also made by the Post Office Research Station for comparison.

4. Results and Discussion

Although both resistance and inductance were determined only the resistance measurements have so far been analysed. A typical

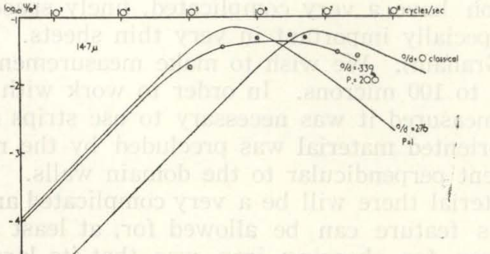

Fig. 1.

result is given in Fig. 1 which shows the quantity U/θ^2 as a function of the parameter $\theta^2 = 2\pi\omega\sigma d^2\mu_0$, μ_0 being the zero frequency permeability. At low frequencies U/θ^2 varies with frequency in reasonably good accord with the domain-and-wall calculations with $\beta = \gamma = 0$ and $a/d = 2.76$. At high frequencies (i.e. when the skin depth becomes much less than the strip thickness) it varies with frequency much more as that predicted by the classical calculations being lower by a roughly constant factor. However, any attempt to fit the high frequency behaviour by suitable choice of the only adjustable parameter, a/d, destroys the agreement at low frequencies. In order to account for the observed behaviour we have investigated the following additional effects.

(a) Spin-relaxation damping of the wall movement.

(b) Domain wall surface tension.

It can be shown that neither of these effects will affect the frequency dependence of U in a way such as to fit more closely the experimental points.

(c) Distribution of domain wall spacing.

It has been shown that, provided the domain wall spacings do not vary over too wide a range (i.e. $\not> 100$) their effect is not significantly different from that calculated using the mean wall spacing as a single parameter.

(d) Intra-domain permeability.

It is almost impossible to assess this precisely but on general grounds it seems likely that the observed behaviour of U at high frequencies is partly attributable to this cause.

(e) Variations in the domain wall restoring force.

It seems highly improbable that all domain walls are equally strongly held in positions of equilibrium; in any real material there will be a wide spread of domain wall restoring force about a mean value. This factor has been investigated in some detail and the theory extended to include it. At low frequencies the more flexible walls make the greatest contribution to the skin-effect which is then determined by the average spacing of the most flexible walls. However, they rapidly become damped out by their own eddy-current field and at higher frequencies more and more walls contribute to the skin-effect. Thus the effect of variable wall stiffness is to give rise to an apparent frequency dependence of the average domain wall spacing; at low frequencies a large wall spacing and at high frequencies a small one.

Calculations were carried out assuming that each stiffness interval within a certain range is equally populated by domain walls, and no walls with stiffness outside this range exist. This form of distribution ensures that most of the domains are strongly held and only very few contribute to the permeability at low frequencies, which seems plausible on physical grounds. In arriving at a value of a/d and the wall stiffness variability factor (P) one must recognize that the choice is to some extent arbitrary, since at a fixed frequency one may choose either a/d or P so as to fit the experimental value. The curve shown in fig. 1 was calculated with $a/d=0.339$ and $P=200$, this choice being to some extent governed by the necessity to put $\tanh n\pi a/d \simeq 1$ in the calculations. It can be seen that the resulting curve fits the experimental points significantly more closely than either the classical curve or the one with a single wall stiffness and with a/d chosen to fit the point at 2 Kc/s. Experimental results taken on strips of other thicknesses showed essentially the same type of behaviour.

These results seem to explain why when eddy-current-loss measurements at low frequencies are used to estimate domain size in thin sheets the domain size obtained comes out to be of the order of the dimensions of a crystal grain (Lee, Lynch, and Eastwood, 1959), whereas powder pattern studies usually show the existence of many domain walls in an average crystallite. Evidently only about one of these, on average, is really mobile.

References

1 J.E.L. Bishop: To appear in Proc. Phys. Soc.
2 E. W. Lee, A. C. Lynch and G. W. Eastwood: Unpublished paper presented at the Sheffield Conference on Magnetism (1959); see R. S. Tebble, and D. E. G. Williams: Brit. J. Appl. Phys. **11** (1960) 307.
3 K. M. Polivanov: Izvestia Akademii Nauk, S. S. S. R. seria Fiziki **16** (1952) 449.

DISCUSSION

W. J. CARR: I am somewhat puzzled by the fact that the distribution in restoring force has no effect at lower frequencies.

E. W. LEE: An effect at low frequencies could be brought about by a very wide distribution of restoring force (which may well exist in many materials) but this does not seem to be required to explain our measurements.

F. J. FRIEDLAENDER: It would appear that the domain wall model proposed to explain the permeability of the iron sheets would be more appropriate if oriented material had been used. We have made measurements on grain oriented 50%Ni-Fe tape cores, using a modification of the method proposed by J. J. Becker and applying a specific domain wall model, we have obtained good agreement with theory. However we also observed that the "effective domain wall area" was different for small signals, as compared with the case of large (irreversible) flux changes.

C. D. GRAHAM, JR: I would like to inquire about the material used in your experimental work. This seems to have been polycrystalline, untextured iron, and if this is true, the ideal domain structure you have assumed will be a very poor approximation to the actual domain structure in the material. Grains whose surfaces are not very nearly parallel to a <100> direction have a very complicated, finely spaced surface domain structure, which will be especially important in very thin sheets.

E. W. LEE (reply to Friedlaender and Graham): We wish to make measurements on sheets whose thickness ranged from 10 to 100 microns. In order to work with a sample whose resistance could easily be measured it was necessary to use strips at least 2 metres long. The use of grain oriented material was precluded by the requirement that the current have a component perpendicular to the domain walls.

I agree with Dr. Graham that in our material there will be a very complicated and fine domain structure. We feel that this feature can be allowed for, at least in principle, by the theory. Our main reason for choosing iron was that its large crystal anisotropy ensures that the intra-domain permeability is small. The theory assumes this permeability to be zero and we think that it is more to try to fulfill this condition than any other.