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Suppression of Spin Wave Instabilities Associated with Ferromagnetic Resonance

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A technique is described which should raise the instability thresholds of spin waves associated with the high power microwave excitation of ferrimagnetic insulators. In particular, it appears possible to suppress instabilities of degenerate spin waves occurring at the main resonance. This is clearly desirable since the resonance could then support larger amounts of power before premature saturation occurred. It also appears possible to raise the threshold of the "subsidiary absorption" and to suppress "parallel pumped" spin waves without simultaneously raising the threshold of certain magnetostatic modes. Limitations of space permit only the stabilization of the main resonance to be considered here; however, a full theoretical treatment covering certain practical applications will be published elsewhere.

Preliminary experimental results, dealing with the suppression of degenerate spin wave instabilities in single crystal YIG, are also given.

One of the present authors[1] has treated the modification of spin wave instability thresholds associated with high power microwave excitation of ferrimagnetic insulators when additional rf excitations, of arbitrary polarizations and frequencies, are present. More recently, theoretical calculations by Suhl[2] and experimental work by Hartwick, Peressini and Weiss[3] have shown that the threshold of certain instabilities may be raised by modulating the dc magnetic field. Spin waves which may be stabilized by this technique include those associated with first order processes[4] (subsidiary absorption) and those driven by "parallel pumping"[5),6),7]. Unfortunately, the Suhl procedure will not stabilize instabilities associated with the main resonance; it also leads to a dissolution of the uniform precession (if present) into many components of different frequencies. Before proposing a remedy, let us examine the appropriate differential equations and understand why one is required.

The differential equations governing the amplitudes and phases of the uniform precession and a single (though arbitrary) spin wave have been obtained for the general case of an ellipsoid, magnetized in an arbitrary direction, when acted upon by an arbitrary number of spatially uniform rf fields[1]. The following equations are specialized for the case of a ferrimagnetic sphere magnetized along the z-direction when the cone angle of precession (θ) is small.

$$\dot{\phi}_0=\omega_0-\frac{1}{\theta}\sum_i \omega_{h_i}\cos(\omega_i t-\phi_0+\alpha_i)+\sum_j \omega_{h_{zj}}\sin(\omega_j t+\beta_j)\,, \qquad (1)$$

$$\dot{\theta}=\sum_i \omega_{h_i}\sin(\omega_i t-\phi_0+\alpha_i)-\omega_{l0}\theta\,, \qquad (2)$$

$$\dot{\phi}_k=\omega_0-\left(\frac{1}{3}-\lambda k^2\right)\omega_M+\omega_M\sin^2\phi\cos^2(\phi_k-\xi)+\sum_j \omega_{h_{zj}}\sin(\omega_j t+\beta_j)\,, \qquad (3)$$

and

$$\delta M=\left\{\frac{\sin^2\phi}{2}\sin(2\phi_k-2\xi)-\theta\frac{\sin 2\phi}{2}\sin(2\phi_k-\phi_0-\xi)+\frac{\theta^2}{2}\left(\lambda k^2-\frac{1}{3}+\cos^2\phi\right)\times\sin(2\phi_k-2\phi_0)-\frac{\Delta H_k}{M}\right\}\omega_M\delta M\,, \qquad (4)$$

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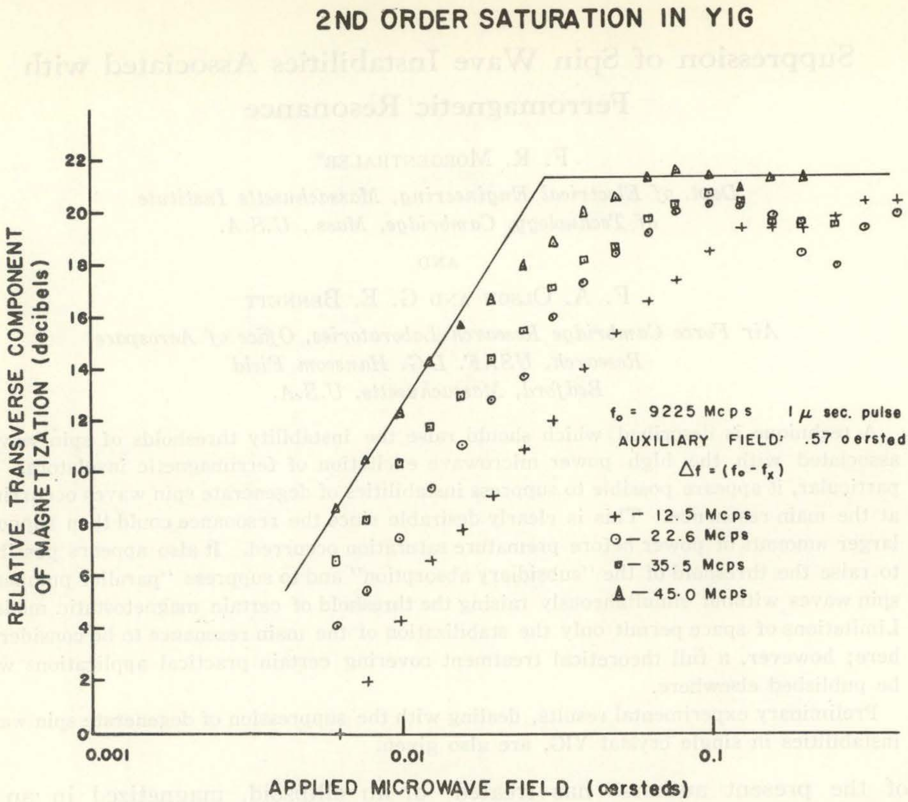

Fig. 1. Second-order saturation of YIG with a transverse modulating field applied.

where ϕ_0 and ϕ_k are the phases of the uniform precession and spin wave respectively, δM is the spinwave amplitude (assumed to be very small), M is saturation magnetization; $\omega_M=-\gamma\mu_0 M$, $\omega_{h_i}=-\gamma\mu_0 h_i$, $\omega_{h_{zj}}=-\gamma\mu_0 h_{zj}$, $\omega_{l0}=-\gamma\mu_0 \Delta H_0$, h_i is the amplitude of a typical circularly polarized rf excitation of frequency ω_i in the transverse plane (positive polarization if ω_i is positive), h_{zj} is the amplitude of a typical longitudinal rf excitation of frequency ω_j, γ is the gyromagnetic ratio (negative) including g factor, α_i, β_j and ξ, are phase constants, $2\Delta H_k$ and $2\Delta H_0$ are the spin wave linewidth and uniform precession linewidth respectively; ψ is the angle between the spin wave propagation vector $\boldsymbol{k}$ and the d.c. field; λ is an exchange parameter.

An instability results if the average value of $\dot{\delta M}$ is greater than zero. This is obtained for spin waves of the proper frequency by increasing θ until either the average value of the second or third terms on the right hand side of Eq. (4) exceeds the spin wave loss. The first term may also overcome the loss independently of θ if "parallel pumping" is utilized.

By inspection of these equations, it is apparent that a longitudinal (z) modulating field causes frequency modulation of *both* ϕ_0 and ϕ_k and hence indirect modulation of θ and δM. It follows that the average value of $\sin(2\phi_k-2\xi)$ and $\sin(2\phi_k-\phi_0-\xi)$ will be reduced by the modulation whereas the average value of $\sin 2(\phi_k-\phi_0)$ will not. The threshold of both "parallel pumped" spin waves and the subsidiary absorption will be effected therefore but not the threshold of the second order coupled spin waves* (in fact because of the indirect modulation of θ, the average threshold cone angle will actually be reduced slightly).

The situation is altered if we choose to

* Actually, for spin waves which are not z-directed ($\psi=0$) the cancellation of the modulation is not complete. Therefore the modulating field may serve to suppress non z-directed spin waves or sharpen the distribution of unstable modes about the z-direction without altering the threshold substantially.

modulate ϕ_0 by using additional transverse excitation[1),8)]. In this case ϕ_k is *not* modulated and the average value of $\sin(2\phi_k-2\phi_0)$ will be reduced; the average value of $(\theta_{\rm crit})^2$ is thereby increased.

Experiments on the suppression of degenerate spin wave instabilities by this last method were carried out by Air Force Cambridge Research Laboratories at X-Band frequencies on single crystal YIG spheres. These consisted of applying to the sample an additional transverse magnetic field h' at a frequency ω' separated by several resonance line widths from the frequency (ω_0) of the field (h_0) driving ferromagnetic resonance. The degree of instability suppression is determined in this case by the strength of modulation which is proportional to h' divided by the frequency difference between the two excitations, $\Delta\omega$. The saturation level was determined as follows. The YIG sphere was located one half wavelength (at ω_0) from the end of a shorted waveguide. A small pickup loop was oriented near the sample in a manner allowing detection of the sample's transverse component of magnetization but minimizing coupling to the waveguide modes[9)]. The transverse component of magnetization, as detected by the loop, was then recorded as a function of h_0 for several values of ω'. These saturation curves are displayed in Fig. 1. The critical field strength for saturation, determined by the usual extrapolation methods, is plotted in Fig. 2 as a function of $\Delta\omega$. The qualitatively correct behavior is apparent.

The actual average resonance cone angle of the extrapolated threshold depends critically on the linewidth of the sample. For small modulation index, ρ, the cone angle is given by

$$\bar{\theta}(\rho)=\frac{h_0}{\Delta H}J_0\left[\frac{h'}{h_0}\frac{\gamma\mu_0\Delta H}{\Delta\omega}\right]$$

$$=\frac{h_0}{\Delta H}J_0(\rho)\,, \qquad (5)$$

where $J_n(\rho)$ is the Bessel function of order n. Using the intrinsic linewidth and the data from Fig. 1, one could calculate an average cone angle as a function of the strength of modulation.

One point which should be stressed is that while the average cone angle may increase slightly with modulation, the excess energy is going mainly into the "side band" modes and not the resonance mode. In other words, we can put additional energy into the sample but the transverse component of magnetization rotating at any given frequency has probably not grown larger than the critical value obtained in the absence of modulation.

Figure 1 tends to confirm this in that all four curves saturate with approximately the same transverse component of magnetization. Furthermore, below saturation the transverse magnetization component is sharply reduced by the presence of the auxiliary field. The reason for this is simply that in order to reduce the coupling strength between the uniform precession and a degenerate spin wave mode we have had to modulate one mode differently from the other. Transverse fields allow us to do this by modulating only the uniform mode but as a price our hoped-for large transverse component of magnetization has dissolved into many small-amplitude side-band components. This is unfortunate since a large transverse response at the driving frequency would be very desirable in many instances. May we reverse this situation by modulating the spin waves and *not* the uniform precession?

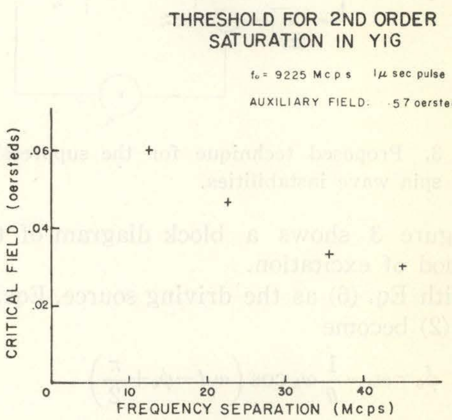

Fig. 2. Dependence of critical field for second-order saturation on $\Delta\omega$ (transverse modulation technique).

The answer, fortunately, is in the affirmative if we utilize *both* transverse and longitudinal modulation in the correct manner*.

Consider the case of the sphere once again,

* The preprint of this paper, written before this fact was discovered, is in error on this point.

and assume that in addition to the dc field along z there exists the following spatially uniform fields:

$$\left.\begin{aligned} h_x &= -h\sin\omega_0 t + h'(t)\cos\omega_0 t\,, \\ h_y &= h\cos\omega_0 t + h'(t)\sin\omega_0 t\,, \\ h_z &= h_z\sin(\omega_z t+\beta_z)\,, \end{aligned}\right\} \quad (6)$$

where

$$h'(t) = \frac{h}{\Delta H}\sin(\omega_z t+\beta_z)\,.$$

The transverse excitation may be obtained by amplitude modulating a portion of the usual positive circularly polarized field which has undergone a phase delay*. The modulation signal must be obtained from the same source providing h_z.

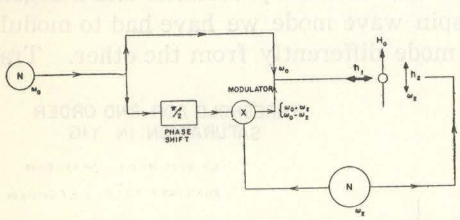

Fig. 3. Proposed technique for the suppression of spin wave instabilities.

Figure 3 shows a block diagram of this method of excitation.

With Eq. (6) as the driving source, Eqs. (1) and (2) become

$$\dot\phi_0 = \omega_0 - \frac{1}{\theta}\omega_h\cos\left(\omega_0 t-\phi_0+\frac{\pi}{2}\right) - \frac{1}{\theta}\omega_{h'}(t)\cos(\omega_0 t-\phi) + \omega_{h_z}\sin(\omega_z t+\beta_z)\,, \quad (7)$$

$$\dot\theta = \omega_h\sin\left(\omega_0 t-\phi_0+\frac{\pi}{2}\right) + \omega_{h'}(t)\sin(\omega_0 t-\phi) - \omega_{l0}\theta\,. \quad (8)$$

The steady-state solution of (7) and (8) is

$$\phi_0 = \omega_0 t\,,$$

and

$$\theta = \frac{\omega_h}{\omega_{l0}} = \frac{h}{\Delta H_0}\,,$$

precisely the solution in the absence of any modulation! In other words we have modulated the uniform precession *twice* in just such a manner that the longitudinal modulation is exactly cancelled by the transverse field. The spin waves, on the other hand, do not see (except in higher order terms) the transverse field so that the longitudinal modulation is not cancelled.

For $\psi=0$ spin waves, Eq. (3) is satisfied by

$$\phi_k = \left[\omega_0 - \left(\frac{1}{3}-\lambda k^2\right)\omega_M\right]t - \delta\cos(\omega_z t+\beta_z)\,, \quad (9)$$

where

$$\delta = \frac{\omega_{h_z}}{\omega_z}\,.$$

It follows that Eq. (4) yields an instability threshold* for the cone angle given by

$$(\theta_{\rm crit})^2 = \left(\frac{h_{\rm crit}}{\Delta H_0}\right)^2 = \left[\frac{2\Delta H_k(k)}{\left(\lambda k^2+\frac{2}{3}\right)MJ_n(2\delta)}\right]_{\min}, \quad (10)$$

where we are to pick that integer value of n (including zero) which minimizes the cone angle. The unstable spin wave has a frequency and wave number given respectively by

$$\omega_k = \omega_0 \pm \frac{n}{2}\omega_z\,,$$

and

$$\lambda k^2 = \frac{1}{3} \pm \frac{n}{2}\frac{\omega_z}{\omega_M}\,.$$

It is important to notice that the frequency of the spin wave of lowest instability threshold

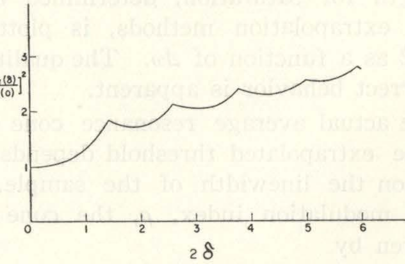

Fig. 4. Critical cone angle as a function of the modulation index.

changes as δ is increased. The sign is also chosen to minimize $\theta_{\rm crit}$; if $n\omega_z/2 \ll \omega_0$ it will generally be positive. In that event

$$\frac{\theta^2_{\rm crit}(\delta)}{\theta^2_{\rm crit}(0)} \simeq \frac{1}{[J_n(2\delta)]_{\max}}\,.$$

This function is shown in Fig. 4.

* If the carrier frequency (ω_0) is suppressed after the modulation takes place, the necessary phase delay is 90°; if it is not suppressed the phase delay will be in the range of 90°—180°.

* It is assumed that $\omega_0/2$ spin waves are forbidden. This is always possible if ω_0 is chosen large enough.

It is apparent that this stabilization procedure allows θ to grow as δ increases. The transverse component of the magnetization, which precesses at the resonance frequency, is proportional to θ (for small θ) and is therefore increased. This is allowed because the stabilizing modulation effects the spin waves without simultaneously modulating the uniform precession.

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DISCUSSION

H. SUHL: I would like to mention that Dr. Dillon of Bell Telephone Laboratories, in connection with an attempt to stabilize the main resonance, suggested simultaneous modulation of field and frequency in order to avoid the sideband problem.