Magnetization Energies and Distributions in Ferromagnetics

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This paper gives a review of some recent theoretical work on magnetostatics. In the first part methods are outlined for the evaluation of the magnetostatic energies of uniformly magnetized bodies. An indication is given of the use of these methods in the treatment of the properties of single-domain ferromagnetic particles, in particular, particles in the form of rectangular blocks and assemblies of such particles, and dendritic particles. The relation between magnetostatic and inductive energies is briefly discussed and the feasibility of determining magnetostatic energies of domain structures by measuring inductances is indicated.

In the second part a method for the calculation of the distribution of magnetization in cylindrical rods is outlined, and it is shown that the results are in good agreement with experiment.

1. Introduction

This paper gives a brief review of some recent theoretical work at Leeds on magnetostatics and its applications. In the first part (§ 2) an account is given of methods for the evaluation of magnetostatic energies of uniformly magnetized bodies and of certain domain structures. Also, an indication is given of the application of the results to the treatment of the properties of singledomain ferromagnetic particles and of assemblies of such particles. In the second part (§ 3) a method is described for the calculation of the distribution of magnetization in cylindrical rods.

2. Evaluation of Magnetostatic Energies

Introduction. Magnetostatic energies play an important part in the determination of the domain structures and low- and mediumfield properties of ferromagnetics, but the calculation of these energies is often difficult. The problems involved are made somewhat more tractable if it can be assumed that the magnetization within a body, or a domain, is uniform. In general, it is only for an ellipsoid that a uniform applied field leads to uniform magnetization and demagnetizing field.¹⁾ Nevertheless, even for non-ellipsoidal bodies there are cases of interest when the magnetization may be treated as uniform, although the demagnetizing field may not be

** Now at Joseph Lucas (Electrical) Ltd., Birmingham. so. Such cases arise, for example, in the consideration of very small particles, and of domains in materials with high magnetocrystalline anisotropy. Although the approximation of uniformity of magnetization is adopted in the methods for evaluating magnetostatic energies discussed below, the results obtained have a much wider bearing.

'Magnetic charge' method

The potential associated with the magnetization of a body may be expressed in the form²⁾

$$V = \int \frac{\sigma}{r} ds + \int \frac{\rho}{r} dv \tag{2.1}$$

where $\sigma = I \cdot n$ and $\rho = -\text{div } I$ are surface and volume 'magnetic charge' densities. The magnetostatic energy of the body, or the mutual energy of two bodies, can then be expressed in terms of the self- and mutual energies of these charges. The use of this method may be illustrated by the following examples.

(i) Rectangular blocks. For a block uniformly magnetized to intensity I_0 parallel to one edge the only 'charges' are uniform surface charges of density $\pm I_0$ on two opposite faces of the block. The magnetostatic energy of the block is then given by the sum of the self-energies of these two surface charges and the (negative) mutual energy between them. Using an analytical expression which has been derived² for the mutual energy of two similarly oriented, uniformly charged rectangular plane areas, it has been possible in this way to calculate the magneto-

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static energies, and hence the 'effective demagnetizing factors'2) and 'shape anisotropy factors'³⁾ of rectangular blocks. The 'effective demagnetizing factor', N, associated with a particular direction in a uniformly magnetized body is defined to be such that the magnetostatic energy per unit volume of body when I_0 lies in that direction is $\frac{1}{2}NI_0^2$. It can then be shown (Rowlands³⁾, Brown and Morrish⁴⁾) that, for a body of arbitrary shape, the sum of the N's associated with any three orthogonal directions is 4π . The 'shape anisotropy factor', ΔN , for a block of square cross-section is defined, by analogy with that for a spheroid, to be the difference between the effective demagnetizing factors along and perpendicular to the long axis of the block. For a single-domain particle in which magnetization changes occur by uniform rotation of the magnetization vector, and in which magnetocrystalline and strain anisotropy are negligible, the coercivity is proportional to ΔN^{3} .

In Fig. 1 the calculated shape anisotropy factors for rectangular blocks are shown as a function of the dimensional ratio, together with those for prolate spheroids⁵⁾ and cylinders^{3) 6)} (see below). Although there are differences in detail between the three sets of results, the general agreement over most of the range suggests that it is a reasonable approximation to treat single-domain particles which are roughly ellipsoidal in shape as rectangular blocks. This often simplifies the theoretical treatment of their properties.



Fig. 1. Shape anisotropy factors, ΔN , of uniformly magnetized rectangular blocks (R), spheroids (E) and cylinders (C) as functions of the dimensional ratio, m. The broken line shows the limiting value, 2π , which all the curves approach as $m \rightarrow \infty$.

(ii) Dendritic particles. By an extension of the method outlined in (i) it has been possible to calculate⁷⁾ the shape anisotropy factors of bodies in the form of rectangular blocks with rectangular 'branches' (see Fig. These approximate in shape to the 2). dendritic particles which are often observed in electron micrographs of small ferromagnetic particles. A typical set of results, for a particle with a 'trunk' with dimensions $(a \times a \times 12a)$ and 'branches' $(a \times a \times 2a)$, is shown in fig. 2. The branches are assumed to be added symmetrically in pairs; particles with 0 and 12 branches correspond to solid rectangular blocks with dimensions $(a \times a)$ $\times 12a$) and $(a \times 3a \times 12a)$, respectively.



Fig. 2. Shape anisotropy factor, ΔN , of a uniformly magnetized dendritic particle as a function of the number of branches, n. The broken lines correspond to the values of ΔN for ellipsoids with the same dimensional ratios as the corresponding blocks. The inset figure shows a particle with n=4.

As may be seen from the figure, the values of ΔN may be considerably smaller than those which might be estimated by considering ellipsoids, either by neglecting the branches or by taking the enclosing ellipsoid. Hence, for a particle in which magnetization changes occur by uniform rotation the coercivity will be correspondingly smaller than that estimated from the dimensions of such ellipsoids.

(iii) *Particle interaction*. By a further extension of the method outlined in (i) calculations have also been made⁷⁾ of the magnetostatic energies of various assemblies of uniformly magnetized rectangular blocks, and the results used in considering the effects of particle interaction on the coercivity of such assemblies. From the detailed calcula-

tions for several particular configurations of blocks it is found that these effects depend strongly on the configuration of the blocks within the assembly, but it is not possible to summarize the results briefly.

Inductance analogy method

It can be shown³⁾ that there are simple relations between (a) the magnetostatic energies of uniformly magnetized bodies, (b) 'magnetic charge energies' and (c) the inductive energies of uniformly wound coils. (An independent derivation of the relation between magnetostatic and inductive energies has been given by Brown⁶⁾.) These provide a means of correlating³⁾ various earlier results of calculations of inductance coefficients and of evaluating magnetostatic energies from those coefficients^{3) 6)}. For example, from tabulated⁸⁾ self-inductance coefficients of cylindrical coils values can be derived for the mutual energy of two uniformly charged coaxial discs as a function of their separation, and these can then be used to evaluate the mutual inductance of two coaxial coils, or to calculate the magnetostatic energies of uniformly magnetized cylinders. The values of ΔN for cylinders shown in figure 1 were found in this way.

In other cases, where the appropriate inductance coefficients have not been calculated, it may be possible to evaluate 'charge energies,' and hence magnetostatic energies of domain structures, by measuring suitable self- and mutual inductances. In order to investigate the feasibility of this method some measurements have been made³ with single-layer coils of rectangular cross-section. It was found possible to obtain 'charge energies' accurate to $\pm 0.3\%$, and the method, therefore, seems practicable. It may be particularly useful in the study of domain structure in small particles containing only a few domains.

3. Distribution of Magnetization in Cylindrical rods

Introduction. As mentioned in § 2, for large-scale specimens it is only for ellipsoids that a uniform applied field leads to uniform magnetization. In many applications of magnetic materials, however, the specimens are not in that form and it is of some importance to be able to calculate the distribution of magnetization within them. As a first step in the investigation of this general problem the particular case of cylindrical rods of material with constant susceptibility has been considered. The results of previous calculations^{9) 10)} do not, in general, agree well with measured values, and a self-consistent method of calculation has now been developed.

Method. For a material with constant susceptibility, κ , the magnetization at any point in the body is given by

$$I = \kappa H_t = \kappa (H_a + H_d), \qquad (3.1)$$

where H_t , H_a and H_d are the total, applied and demagnetizing fields at that point. Also,

$$\operatorname{div} \boldsymbol{B} = \operatorname{div}(\boldsymbol{H}_t + 4\pi \boldsymbol{I}) = 0 \tag{3.2}$$

Then, for a cylinder in a uniform, axial applied field, writing $H_d = -\text{grad } V$ and using cylindrical polar coordinates,

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{\partial^2 V}{\partial z^2} = 0.$$
 (3.3)

Using (3.3), it can be shown that the potential at any point in the cylinder, V(z, r), is related to that on the axis, V(z, 0), by

$$V(z, r) = \sum_{k=0}^{\infty} \left\{ \frac{(-1)^k}{2^{2k} (k!)^2} \frac{d^{2k} V(z, 0)}{dz^{2k}} \right\} r^{2k}.$$
 (3.4)

In the present work V(z, 0) has been expressed as a terminating power series of the form

$$V_0(z, 0) = \sum_{j=0}^{n} c_{2j+1} z^{2j+1}.$$
 (3.5)

From (3.1)–(3.5), H_d and hence I, at any point can be expressed in terms of the coefficients, c_{2j+1} . Then V(z, 0) can be recalculated by means of (2.1) to give, say, $V_1(z, 0)$. The values of the c_{2j+1} are then chosen so as to minimize the difference between $V_0(z, 0)$ and $V_1(z, 0)$. In the present work the quantity minimized was $\sum_i [V_0(z_{i,0}) - V_1(z_{i,0})]^2$, where the z_i correspond to 21 equi-spaced points on a semi-axis of the cylinder.

Potential functions have been determined⁷) by this method for cylinders with dimensional ratios in the range 1–500 and susceptibilities in the range $0.1-5\times10^4$. The calculations were made with the aid of an electronic computer, and the number of terms used in (3.5) was in the range 3–12. From these potential functions a variety of properties of the cylinder can be calculated. As an example, the distribution of the axial component of magnetization of a rod with dimensional ratio 50 is shown in Fig. 3 for several values of susceptibility. Also in Fig. 3 are some experimental results due to Bozorth and Chapin¹¹⁾ for an unannealed permalloy rod with approximately constant susceptibility. The agreement between the calculated and measured variations is good, and is much better than that found using the results of previous calculations.

The method outlined above can also be extended to apply to permanently magnetized



Fig. 3. The axial component of magnetization of a cylindrical rod, with dimensional ratio 50, as a function of distance from the centre along the axis. The numbers on the curves show the values of the susceptibility. The broken curve shows the measured variation for a rod with dimensional ratio 52.4 and susceptibility approximately 6.5 (Bozorth and Chapin⁽¹¹⁾). $\zeta = z/a$, where 2a is the length of the rod. cylinders and to cylinders in which the susceptibilities in the axial and radial direction are not equal.

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DISCUSSION

W. F. BROWN: 1. I congratulate Dr. Rhodes and his colleagues on their systematic handling of the cylinder of constant permeability. As far as I know the only rigorously solved problems for bodies of constant permeability in a constant applied field are the ellipsoid, the limiting case of zero susceptibility, and the limiting case of infinite susceptibility for the transversely magnetized infinite rectangular bar.

2. I am glad to see magnetic charges (poles) and equivalent currents used on the basis of convenience rather than of some supposed greater significance of one or the other. Text book writers and pedagogues have misled us in this respect by their over-enthusiasm, in the last decade or two, for Amperian currents as against poles.

3. Coils can be used as analogs not only to pole distributions, but to mass distributions as sources of gravitational fields. This principle has applications in geophysical prospecting.

K. HOSELITZ: The authors have calculated the demagnetizing factors for particles with dendrites. One can use their data to estimate the coercive force of uniformly magnetized particles such as those produced by the E.S.D process at the General Electric Research Laboratories, Schenectady. How does the present calculation compare with the observed coercive forces, which can quite easily be explained by the process of non-uniform demagnetization (for instance fanning with the chain of sphere model). Is there any criterion such as particle interaction which allows us to decide whether the changes in magnetization are uniform or non-uniform, since the coercive force can now be explained by their model?

P. RHODES: For sufficiently small particles magnetization changes will take place by uniform rotation and the coercivity is then determined directly by the calculated shape anisotropy factor.

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Magnetic Image Contrast in Electron Mirror Microscopy

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The feasibility of electron mirror microscopy for visual observation of magnetic stray fields on surfaces will be demonstrated. Magnetic image contrast as well as the criteria which permit the discrimination of magnetic patterns from patterns of other origin will be briefly discussed. Electron mirror micrographs of domain patterns and of magnetic patterns recorded on magnetic tapes and films will be shown. A motion picture depicting magnetic domains in motion as well as emerging magnetic stray fields on grain boundaries in silicon iron will also be presented.

Electron optical methods for visual observation of magnetic domains have been introduced only rather recently. This is somewhat surprising because even conventional electron transmission microscopy can be utilized for this purpose as Hale, et al.1, Boersch, et $al^{(2)}$ and others³⁾ have shown. Transmission microscopy requires, of course, specimens thin enough to be penetrated by electrons. If this is not the case, one must rely on electron optical methods which depict surfaces. Electron emission microscopy can then be used as an instrument for the observation of magnetic domains as Spivak, et al.4) have shown. Another possibility is electron mirror microscopy⁵⁾ which can be utilized not only for the visual observation of the distribution of such electrical properties as surface potentials, conductivities, etc., but serves equally well in depicting magnetic domain patterns⁶⁾ and artificial magnetic patterns recorded on magnetic media⁷⁾.

In electron mirror microscopy the specimen constitutes an electron optical mirror biased slightly negative with respect to the electron source, i.e., the cathode. The electrons therefore do not reach the mirror-specimen but are reflected at the zero equipotential which is to be located close enough to the specimen to carry the geometric relief and the magnetic or electrical relief of the specimen proper. In the magnetic case the image contrast forming deflection is caused by that component $(F = e\dot{r}B_z)$ of the Lorentz force **F** which stems from the interaction of the magnetic field normal to the plane of the mirror-specimen (B_z) with the radial component of the electrons' velocity (\dot{r}) . The sensitivity of electron mirror microscopy to magnetic fields is therefore zero at the electrical center of the mirror-specimen, i.e., where the radial component of the electrons' velocity becomes zero. This feature, which might appear detrimental, is actually advantageous because it establishes a convenient criterion for distinguishing patterns of magnetic origin from patterns which stem from the geometrical relief structure or from those which are of electrical origin. The procedure for determining whether a pattern is of magne-