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A New Method for the Determination of the Ferromagnetic Exchange Constant

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A new method for the determination of the ferromagnetic exchange constant A is described. It is based on the response of the magnetization to an inhomogeneous magnetic field, which is generated by passing a current through a thin ferromagenetic sample. The theory is given for the two cases of a plane film and a cylindrical wire. Preliminary experimental results on a 6600 Å permalloy film gave a value of $A=0.55\times10^{-6}$ erg/cm$\pm10\%$.

Spatial gradients of the magnetization $\boldsymbol{M}$ in a ferromagnet are associated with an exchange density

$$E_{ex}=\frac{A}{M^2}[(\nabla M_x)^2+(\nabla M_y)^2+(\nabla M_z)^2]\ . \tag{1}$$

The exchange constant A can be determined from the Bloch $T^{3/2}$ law[1),2)] and from spinwave measurements at microwave frequencies[3),4)]. Both methods employ the dispersion relation $\omega=\gamma(H_0+(2A/M)k^2)$, the first one for the thermal spinwaves, the second one for the spinwaves excited by the microwave field. A further possibility is the determination of A from measurements of the Bloch wall energy $\gamma_w=4\sqrt{AK}$[5),6),7)].

In the method described in this paper, the exchange constant is determined by comparing the characteristic length $\delta=\sqrt{A/K}$ with the geometrical extension of a thin ferromagnetic sample. The method is based on the response of the magnetization to an inhomogeneous magnetic field, which is generated by passing a current through the sample.

Theory

We consider a ferromagnetic film in the (x, y)-plane, uniformly magnetized along a direction of easy magnetization, which we take as the x-axis (Fig. 1). Since we are only interested in small deviations φ of the magnetization $\boldsymbol{M}$ from the easy direction, the details of the anisotropy (uniaxial, cubic, etc.) do not enter into the calculations and we may write for the anisotropy energy

$$E_K=\text{const.}+K\varphi^2\ . \tag{2}$$

We can influence the effect of the anisotropy by a constant homogeneous field H_x.

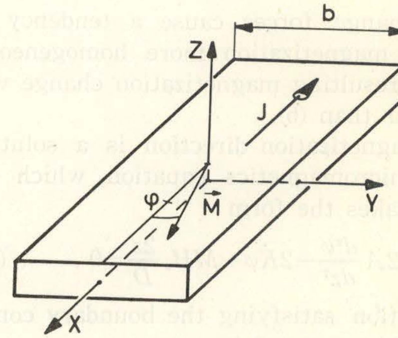

Fig. 1.

This can be taken into account by replacing the anisotropy constant K and the anisotropy field $H_K=2K/M$ by

$$\left.\begin{aligned}\hat{K}&=\frac{1}{2}MH_x+K\ ,\\ \hat{H}_K&=H_x+H_K\ .\end{aligned}\right\} \tag{3}$$

The case of the x-axis being a hard direction may be included by allowing K and H_K to assume negative values, provided $H_x>-H_K$.

If such a film is subjected to a homogeneous field H_y, the exchange forces have no effect, and the response to a small field is given by

$$\left.\begin{aligned}\frac{M_y}{M}&=\varphi=\frac{H_y}{\hat{H}_K}\ ;\\ \varDelta_{\text{hom}}&=\frac{M-M_x}{M}\\ &=\frac{1}{2}\varphi^2=\frac{1}{2}\left(\frac{H_y}{\hat{H}_K}\right)^2\ .\end{aligned}\right\} \tag{4}$$

In order to observe exchange effects, an inhomogeneous field must be applied. This can be produced most conveniently by passing a current through the sample. For a current J parallel to the x-direction of the film with thickness D and width b, the field is

$$\left.\begin{aligned} H_y &= H_0 \frac{2z}{D}\,; \\ H_0 &= 0.4\pi J/2b\,. \end{aligned}\right\} \quad (5)$$

If no exchange forces were present, the local magnetization direction would still be given by (4), resulting in a change of the average magnetization along the x-direction of

$$\Delta_{\text{inhom}} = \frac{M - \langle M_x \rangle}{M} = \frac{1}{2}\langle \varphi^2 \rangle = \frac{1}{6}\left(\frac{H_0}{\hat{H}_K}\right)^2 . \quad (6)$$

The exchange forces cause a tendency to keep the magnetization more homogeneous, and the resulting magnetization change will be smaller than (6).

The magnetization direction is a solution of the micromagnetics equation, which for small φ takes the form

$$2A\frac{d^2\varphi}{dz^2} - 2\hat{K}\varphi + MH_0\frac{2z}{D} = 0\,. \quad (7)$$

The solution satisfying the boundary conditions $(d\varphi/dz)|_{z=\pm D/2} = 0$ is

$$\varphi = \frac{H_0}{\hat{H}_K}\left[\frac{2z}{D} - \frac{2\hat{\delta}}{D}\sinh\frac{z}{\hat{\delta}} \Big/ \cosh\frac{D}{2\hat{\delta}}\right], \quad (8)$$

with $\hat{\delta}$ defined as

$$\hat{\delta} = \sqrt{\frac{A}{\hat{K}}} = \sqrt{A \Big/ \left(K + \frac{1}{2}MH_x\right)}\,. \quad (9)$$

The resulting change in the average magnetization is

$$\Delta_{\text{inhom}} = \frac{M - \langle M_x \rangle}{M} = \left(\frac{H_0}{\hat{H}_K}\right)^2 F\left(\frac{D}{2\hat{\delta}}\right), \quad (10)$$

where $F(D/2\hat{\delta})$ is the function

$$F(\xi) = \frac{1}{6} - \frac{1}{4\xi^2} \times \left[4 - \frac{5}{\xi}\tanh\xi + \frac{1}{\cosh^2\xi}\right]. \quad (11)$$

A result analogous to (10) has been obtained for the infinite cylinder with a function $C(D/2\hat{\delta})$ which is given by

$$C(\xi) = \frac{1}{4} - \frac{2}{\xi^2[I_0(\xi) + I_2(\xi)]^2} \times [3I_0(\xi)I_2(\xi) - I_1^2(\xi) + 2I_2^2(\xi)]\,, \quad (12)$$

where the $I_n(\xi)$ are the Bessel functions of imaginary arguments, $I_n(\xi) = i^{-n}J_n(i\xi)$, and D is now the cylinder diameter. The functions $F(D/2\hat{\delta})$ and $C(D/2\hat{\delta})$ are plotted in Fig. 2.

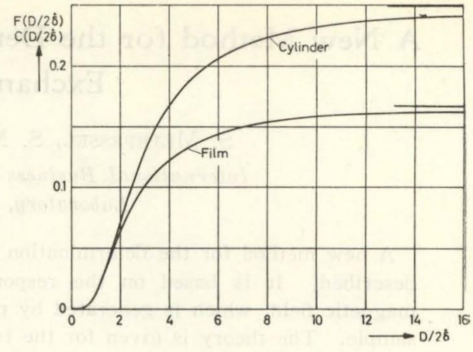

Fig. 2.

Expansions valid for small and large x are

$$\left.\begin{aligned} F(\xi) &= \frac{17}{630}\xi^4 \quad &\text{for} \quad \xi \ll 1 \\ &= \frac{1}{6} - \frac{1}{\xi^2} + \frac{5}{4\xi^3} \quad &\text{for} \quad \xi \gg 1 \end{aligned}\right\} \quad (13)$$

and

$$\left.\begin{aligned} C(x) &= \frac{11}{512}\xi^4 \quad &\text{for} \quad \xi \ll 1 \\ &= \frac{1}{4} - \frac{2}{\xi^2} + \frac{5}{2\xi^3} \quad &\text{for} \quad \xi \gg 1 \end{aligned}\right\} \quad (14)$$

Experiments

If H_0 and H_y are *ac* fields of frequency f, the magnetization changes Δ_{inhom} and Δ_{hom} can be determined from the voltages U_{inhom} and U_{hom} which are induced with frequency $2f$ in a pickup coil with its axis parallel to x. One obtains* from (4) and (10)

$$F\left(\frac{D}{2\hat{\delta}}\right) = 2\left(\frac{b}{0.4\pi}\right)^2 \frac{U_{\text{inhom}}}{U_{\text{hom}}} \frac{H_y^2}{J^2}\,. \quad (15)$$

For the measurement, a film evaporated onto a long glass strip with its easy direction parallel to the long side of the strip was placed between two pairs of Helmholtz coils which produce H_y and H_x. The output signal from a pickup coil wound around the center of the strip was amplified by a resonance amplifier tuned to $2f$ and measured by a vacuum tube voltmeter. The pickup coil was adjusted in such a way that no air flux coupling from the *ac* field H_y and the *ac* current J occurred, *i.e.*, that the output signal is zero if the magnetization is fixed by a

* It is assumed that the frequency f is small compared with the resonance frequency, so that the static theory is still valid.

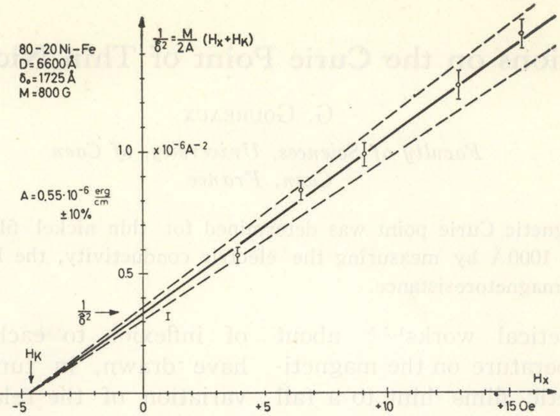

Fig. 3.

large *dc* field H_x.

The output signals U_{inhom} and U_{hom} were measured as functions of J^2 and H_y^2, respectively, with several H_x values as parameter, and the proportionality to J^2 and H_y^2, according to (4) and (10) was established. The value of the function $F(D/2\hat{\delta})$ was calculated from (15). From Fig. 2 and the thickness D of the film (measured by the Tolanski method), one value $\hat{\delta}$ is obtained for each value of H_x.

According to (9) a plot of $1/\hat{\delta}^2$ versus H_x is expected to be a straight line, which intersects the H_x axis at $H_x = -H_K$. From the slope of this line, the exchange constant A can be determined.

Fig. 3 shows as an example the results obtained at $f=1$ kcps for an 80-20 Ni-Fe film, evaporated onto a 5 cm long glass strip, with $b=0.85$ cm; $D=6.6\times10^{-5}$ cm; $H_K=4.7$ oe; $M=800$ gauss. From the slope of the line, a value of $A=0.55\times10^{-6}$ erg/cm$\pm10\%$ is obtained for the exchange constant of this film.

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DISCUSSION

P. E. Tannenwald: I should like to ask you to compare the gradient of the inhomogeneous magnetization in your experiment with that in which Rado and coworkers measure exchange effects.

H. Thomas: The relative field variation $1/H_0 \cdot dH/dx$ in our experiment is of the order of the reciprocal film thickness and therefore somewhat smaller than the corresponding quantity in the experiments of Rado and coworkers.

G. T. Rado: I believe you mentioned that your experiment does not depend on the boundary condition at the film surface. It seems to me, however, that this boundary condition may play a role in your experiment provided the sign of the surface anisotropy is such that the spins at the surface are normal to the surface. In this case a "wall"-type region of spin orientations should exist near the surface.

H. Thomas: Our analysis strictly applies only to those values of the surface anisotropy which leave the film almost in the single domain state *i.e.* which create at most small deviations of the surface magnetization from the easy axis of the film. In this case, the boundary condition is independent of the surface anisotropy. In the opposite case, namely that in which the film normal is an easy axis of surface anisotropy with a very high anisotropy constant, the problem is more complicated, because the surface region may break up into a domain like structure in order to reduce stray field energy. Such a surface domain structure was, however, never observed on our films.