# On Domain Walls Inclined to the Easy Axis in Thin Ferromagnetic Films

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The paper shows how the theory of the hysteresis loop in uniaxial thin films may be applied to predict the angle between the domain walls and the direction of the applied field, as function of the angle between the field and the easy axis.

Bitter patterns of the region between the main part of the film and nuclei of reverse magnetisation remote from edges show a zig-zag boundary, and a characteristic part of this boundary gives good agreement with the theory.

### 1. Introduction

The result of applying a field inclined at an angle  $\theta$  to the easy axis of a thin uniaxial film, Fig. 2 below, is to swing the magnetisation direction through an angle  $\alpha$ which can be calculated in the usual way<sup>1).2)</sup>.

If the anisotropy and the magnetisation energy are the only important energy terms:

 $E = E_a + E_m = K \sin^2 \alpha + MH - MH \cos (\theta - \alpha)$ , then the equilibrium position of the magnetisation vector is obtained from the energy minimum when  $dE/d\alpha = 0$ . That is, when the normalised applied field h is related to  $\theta$  and  $\alpha$  by

$$h = \frac{MH}{2K} = \frac{\sin 2\alpha}{2\sin(\theta - \alpha)} \tag{1}$$

The hysteresis loop is then found by plotting  $|i| = \cos(\theta - \alpha)$ , which is the component of the normalised intensity of magnetisation parallel to the applied field, against |h|, with  $\theta$  as parameter. Curves like those of Fig. 1 show an unstable region when the magnetisation abruptly changes direction at a critical value of applied field. Such loops show excellent agreement with measurement<sup>1),2)</sup>.

# 2. Theory of the Critical Angles of the Domain Walls

Consider the region of instability in the theoretical loop in more detail. Fig. 1 shows the loop for the case of  $\theta = 60^{\circ}$ , Fig. 2 the vector diagrams in the film at successive instants starting at remanence (P) and approaching the unstable point (A).

As h increases from zero the magnetisation direction swings round until the point A is reached when it jumps to B. The process is sketched in both figures. As |h| increases beyond  $|h|_{A}$ , |i| moves along the saturated limb in Fig. 1, and the direction of the magnetisation swings towards that of h,  $(\theta - \alpha)$ 



Fig. 1. Theoretical hysteresis loop for  $\theta = 60^{\circ}$ .



Fig. 2. Vector diagram showing successive directions of the magnetisation.

diminishing, Fig. 2.

The value of  $\alpha$ ,  $\alpha_A$ , at which the discontinuous jump occurs can be found from the fact that at A (Fig. 1) d|h|/d|i|=0. Provided  $\theta \neq 0$  or  $\theta \neq \alpha$ , this condition is equivalent to  $d|h|/d\alpha=0$ .

This leads to

$$\tan \alpha_A = (\tan \theta)^{1/3} . \tag{2}$$

The value of  $\alpha_B$  is found from the intersection of  $|h| = |h_A|$  with the upper limb of the hysteresis loop. Thus the critical angles between the magnetisation direction and the easy axis can be found for all values of the direction of the applied field.

### 3. Application to a 'Fine Wire' Experiment

Apply this theory to a thin film in which the component of magnetisation in the plane of the film is set up by current in a fine wire lying close to the film surface, Fig. 3.



Fig. 3. Arrangement for fine wire experiment.

This gives a uniformly varying h over the film, and in certain regions, h will reach the critical value  $h_A$ , the magnetisation will jump to state B and a domain of reverse magnetisation will form there, separated by a domain wall from the remainder of the film.

## 4. Relation between the Domain Wall Angle and the Critical Angles $\alpha_A$ , $\alpha_B$

Whether or not there is some free pole density in the domain wall, the wall very closely will bisect the angle between the magnetisation directions  $\alpha_A$ ,  $\alpha_B^{30}$ . (It is usual to take the experimental fact that domain walls do bisect this angle as evidence that div I=0 in the walls<sup>4)</sup>. However appreciable free pole density causes negligible deviation from the bisecting lines.) From this fact, and from equation 2, the values of the angle  $\phi$  between the domain wall direc-



Fig. 4. Graph of angles  $\alpha_A$ ,  $\alpha_B$ ,  $\varphi$ ,  $\psi$  against  $\theta$ .

tion and the applied field direction may be found as  $\theta$  varies, Fig. 4.

Note that when  $\theta$  is large, the angle  $\phi$  may exceed 90°, i.e., that the slope of the wall relative to the direction of the easy axis reverses. At  $\theta \doteq 76\frac{1}{2}^{\circ}$  the wall lies at 90° to the applied field and parallel to the wire.

### 5. Relation between Domain Wall Angle and the Easy Direction

In a similar way one may obtain the angle  $\Psi$  between the domain walls and the easy axis of magnetisation in the film, Fig. 4, as the direction of applied field varies. It turns out that the angle between the domain walls and the easy axis is always less than  $15^{\circ}$ .

### 6. Experimental Confirmation

The 'fine wire' experiment was carried out and Fig. 5 a-e shows typical results for a nickel iron film (composition *nominally* 79:21). The domain patterns were obtained using a highly sensitive colloid<sup>5</sup>) which was capable of following slow movements of the pattern. To minimise the effect of local variations of the direction of the easy axis in the film, material of zero magnetostriction was selected. This was established by subjecting the film substrate to a bending moment and verifying that the B-H loop was unaffected.

Only the regions XX shown in Fig. 6 behave in accordance with this theory. And in these regions the angles are as predicted to within the experimental error of  $\theta$  ( $\pm 2^{\circ}$ ). In particular when  $\theta = 75^{\circ}$  (Fig 5d) these regions run nearly parallel to the wire. When  $\theta = 80^{\circ}$  (Fig. 5e) the slopes of part of the boundary should reverse, and in places they appear to do so.

The remaining regions of the domain boundary cannot be explained by this theory. However if the existence of free poles in the







Fig. 5. Bitter Patterns showing the actual and predicted directions of part of the domain walls. The black bar is the wire, diameter  $125 \mu$ .



wall is postulated the other regions of the boundary can also be explained, in considerble detail<sup>3)</sup>.

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Fig. 6. Regions XX of domain wall at angles obeying the theory.

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