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Statistical Theory of Coercive Field

H. D. DIETZE

Technische Hochschule, Aachen and Kernforschungsanlage, Jülich, Germany

Following an idea by Néel a consistent theory of the coercive field is developed, the results of which are given here. These results are applied to experiments of Qureshi.

In 1946 Néel¹⁾ published a general theory of the coercive field. In this theory the fundamental assumption is made that the defects are randomly distributed in the crystal and that every domain contains a great number of defects. In elaborating the theory Néel has to introduce very many special assumptions. Especially a certain interaction potential between the defects and the walls has to be postulated, which is not realized in nature. In fact the resulting formula apparently has never been applied by Néel to any special cases.

The author succeeded in developing, without special assumptions, a statistical theory of the coercive field, the result of which can be applied to a concrete case. In this paper the general presumptions and the result of the theory shall be reported.

The following assumptions were made:

1. The defects are randomly distributed in the crystal and every domain contains a large number of these defects. The density of defects of a certain type l is called n_l .

2. A single defect of type l has an interaction energy $\phi_l(x)$ with the wall (x: distance of the defect from the wall).

3. The interaction energies of the defects are independent of each other, *i.e.*, the total interaction energy is the sum of the interaction energies of the single defect.

The calculations are carried out by means of a statistical method similar to that applied by Néel, however, without any further special assumptions. The bending-out of the walls is taken into account as in Néel's theory. The formula for the coercive field is

$$H_{s} = \frac{1}{12} \frac{1}{J_{s}\eta\bar{\gamma}} \sum_{l} n_{l}$$
$$\times \int_{-L/2}^{L/2} \left(\frac{d\phi_{l}}{dx}\right)^{2} dx \left(\frac{1}{5}\ln\frac{L}{2\eta}\right)^{1/2}. \quad (1)$$

 J_s : saturation magnetization

 $\bar{\gamma}$: average wall energy

L: distance between the walls

Here the correlation length η introduced by Néel is defined by

$$\eta = \left(\frac{2\sum_{l} n_{l} \int_{-L/2}^{L/2} \left(\frac{d\phi_{l}}{dx}\right)^{2} dx}{\sum_{l} n_{l} \int_{-L/2}^{L/2} \left(\frac{d^{2}\phi_{l}}{dx^{2}}\right)^{2} dx}\right)^{1/2} .$$
(2)

A detailed presentation of the theory will be published in Z.f. Physik.

Qureshi²⁾ measured the coercive field in iron with precipitated copper. He used alloys with 0.9, 1.6, and 2.14% Cu which were annealed for 12 hours in hydrogen at 875°C and then quenched. In this state all of the copper is in solution. Let H_{co} be the coercive field of this state. The samples were then annealed for various times at 690°, 710°, and 730°C. During the first minutes of the anneal treatment, when the coercive field has not yet changed, the copper content above the boundary of solubility is precipitated quantitatively. For the different states Qureshi determined the coercive field H_c (at R.T.) and (with an electron microscope) the radii R of the copper precipitates. Fig. 1 shows the experimental results for a sample with 2.14% Cu and an anneal temparature of 710°C.



Fig. 1. Experimental curve and theoretical wall energy effect curve of the coercive field versus the radius of the precipitates.

The radii of the copper precipitates are given in multiples of the wall thickness

$$\delta = \left(\frac{A}{K_1}\right)^{1/2} \approx 1.6 \cdot 10^{-6} \,\mathrm{cm} \,(\mathrm{R.T.}) \,.$$

A: exchange energy constant,

 K_1 : crystal anisotropy constant

The cause for the coercive field H_{co} in the perfect solution is not known. It is assumed that the statistical theory can be applied also to H_{co} and that the correlation length η is but slightly dependent on the state of the sample. In this case, according to eq. (1), the total coercive field is the sum of the coercive field due to the copper precipitates and H_{co} . The interaction potential ϕ for a copper precipitate is the sum of a part ϕ_w which is equal to the wall energy associated with the hole and a part ϕ_s due to the magnetic stray field. Up to now only ϕ_w could be calculated. The part $H_c(w)$ due to ϕ_w is shown in Fig. 1. The calculation of ϕ_s presents serious difficulties as the magnetic field due to the poles at the surface of the precipitate changes the magnitization in the interior of the material *i.e.* poles are induced in the interior. In the moment the calculations are under way.

According to eq. (1) the coercive field may be written

$$H_{o}-H_{co} = \frac{1}{12} \frac{n}{J_{s}\eta\bar{\gamma}} \left[\int \left(\frac{d\phi_{w}}{dx}\right)^{2} dx + 2 \int \frac{d\phi_{w}}{dx} \frac{d\phi_{s}}{dx} dx + \int \left(\frac{d\phi_{s}}{dx}\right)^{2} dx \right] \cdot \left(\frac{1}{5} \ln \frac{L}{2\eta}\right)^{1/2}.$$

Here

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$$\int \left(\frac{d\phi_w}{dx}\right)^2 dx = VK_1 J_s^2 F_w(\rho) ,$$

$$2\int \frac{d\phi_w}{dx} \frac{d\phi_s}{dx} dx = V J_s^4 F_{ws}(\rho) ,$$

$$\int \left(\left(\frac{d\phi_s}{dx}\right)^2 dx = V J_s^6 K_1^{-1} F_s(\rho) .$$

with $\rho = R/\delta$ and $\delta = (A/K_1)^{1/2} \sim J_s K_1^{-1/2}$.

V is the volume of a single precepitate. F_w, F_{ws} , and F_s are functions of ρ only and are dependent on temperature only through ρ . Because $\bar{\gamma} \sim J_s K_1^{1/2}$ one obtains

$$H_{c} - H_{co} = \text{const.} \frac{v}{\eta} \\ \times [K_{1}^{1/2} F_{w}(\rho) + J_{s}^{2} K_{1}^{-1/2} F_{ws}(\rho) \\ + J_{s}^{4} K_{1}^{-8/2} F_{s}(\rho)], \quad (3)$$

v=nV is the ratio of the volume of the precipitates and the total volume.

The first term of eq. (3) is plotted in Fig. 1. In one case is η assumed to be constant and in the other case $\eta(\rho)$ is calculated according to eq. (2) taking into account only ϕ_w .

We are now going to check the linear dependency of the coercive field on v in eq. (3). In Fig. 2 $(H_c - H_{co})$ is plotted versus the copper concentration for an anneal temperature of 730°C and different values of ρ .



Fig. 2. Coercive field versus copper concentration for various precipitate radii.



Fig. 3. Temperature dependence of the coercive field.

The solubility determined from resistance measurements is 0.65%. Finally one may check in a qualitative manner the temperature dependence of the coercive field. The experimental result is given in Fig. 3. As $\rho = R/\delta$ decreases, for a fixed R, with increasing temperature, the functions $F_w(\rho)$, $F_{ws}(\rho)$, and $F_s(\rho)$ will decrease with increasing temperature. In eq. (3) F_w is multiplied by $K_1^{1/2}$ so that the wall energy effect will decrease rapidly with temperature. The factor $J_s^4 K_1^{-3/2}$ by which F_s is multiplied periments prior to publication and to Prof. increases, however, rapidly with temperature which probably explains the peculiar temperature dependence of the coercive field in Fig. 3. The temperature dependence of η cannot be deduced as η is dependent also on those causes which lead to H_{co} .

My thanks are due to Dr. A. H. Qureshi for informing me of the results of his exDr.-Ing. M. Kersten for many stimulating discussions.

References

1 L. Néel: Annales de l'Université de Grenoble 22 (1946) 299.

2 A. H. Qureshi: Z. f. Metallkunde, 52 (1961) 799.

DISCUSSION

C. D. GRAHAM: It seems to me that you have a serious experimental problem in checking your theory. How are the Cu precipitate particle diameters measured to three significant figures in the range of 1000Å, and are the precipitates uniform in size so that it is sufficient to use one value of R?

H. D. DIETZE: The size of precipitate particle is measured by electron microscope. The precipitates are not uniform in size.

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The Electron Microscope Examination of Small Ferromagnetic Precipitates*

P. GAUNT

Department of Physics, the University of Sheffield Sheffield, 10, England

AND

J. SILCOX**

Cavendish Laboratory, the University of Cambridge Cambridge, England

Small particles of cobalt precipitated from a Au 1.5% Co alloy and from a Cu 2.4% Co alloy have been examined using the thin film electron microscope technique. In the gold alloy the precipitates form in disc-like arrays on {100} planes of the gold matrix. In the copper alloy the particles are thought to be roughly spherical. It is suggested that the high coercivities observed in the gold alloy cannot be explained satisfactorily in terms of particle shape and particle interaction alone, but that particle shape and particle interaction could provide the main contribution to the lower coercivity of the copper alloy.

Introduction

The shape, size and distribution of small ferromagnetic particles precipitated from

** Now at Department of Engineering Physics, Cornell University, Ithaca, New York, U.S.A.

non-ferromagnetic matrices are currently being examined using the thin film technique in order to correlate these observations with the magnetic properties of such materials.

Gold 1.5% Cobalt Alloy

Some results have already been reported on

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^{*} Read by W. Sucksmith.