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V-9. Electron Effective Mass in Gallium Arsenide as a Function of Doping

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Faraday rotation measurements have been made on GaAs samples having carrier concentrations between 2×10^{14} and $7.1\times10^{18}\,cm^{-3}$ in the wavelength region between 0.9 and 6.5 microns at room and liquid nitrogen temperature. These measurements are interpreted by considering the effect of multiple internal reflection and related depolarization effects for incoherent radiation in terms of the optical constants. The best agreement of theory with the experimental data is obtained for an effective mass of $m_0^*/m_e=(.066\pm.002)$ at the bottom of the GaAs Γ_{1c} conduction band.

§1. Introduction

Free-carrier Faraday rotation is used to determine the electron effective mass in semiconductors with high carrier concentrations. However, to interpret Faraday rotation measurements, information is needed on carrier concentrations and optical constants. The carrier concentrations are usually obtained from Hall effect measurements; and the optical constants are generally determined by reflection and transmission experiments.

§2. Theory

(a) Band model

For low magnetic fields under the condition $(\omega-\omega_c)\tau\gg1$ the Faraday rotation for free electrons is given by the formula[1)]

$$\theta=\frac{e^3BN\lambda^2d}{8\pi^2c^3\varepsilon_0nm^{*2}}, \tag{1}$$

where d is the sample thickness, ε_0 is the permittivity of vacuum, n is the refractive index in the absence of a magnetic field, and m^* is the effective mass of the carriers. The plane-polarized radiation is incident normal to the sample and the magnetic field is parallel to the Poynting vector.

The nonparabolic character of the energy bands leads to an effective mass variation with carrier concentration.[2,3)] For E_F, $\Delta\ll E_g$, the energy dependence of the effective mass is given by

$$\frac{1}{m^*}=\frac{1}{m_0^*}\left[1-\frac{10kT}{3E_g}\left(\frac{F_{3/2}}{F_{1/2}}\right)\right], \tag{2}$$

where E_F is the Fermi energy determined by the doping level, Δ is the spin orbit splitting, m_0^* is the effective mass at the bottom of the conduction band, k is the Boltzmann constant, T is the absolute temperature, and $F_{3/2}$ and $F_{1/2}$ are the Fermi integrals.[4)] Since lattice vibrations are ignored in the derivation of eq. (2), only the dilational change in the energy gap E_g should be introduced in calculating the temperature dependence of m^*/m_0^*.[14,7)] The Fermi energy can be determined from the following expression for the carrier concentration N:

$$N=\frac{\sqrt{2}(kT)^{3/2}m_0^{*3/2}}{\pi^2\hbar^3}\left[F_{1/2}+\frac{5kT}{2E_g}F_{3/2}\right]. \tag{3}$$

(b) Effect of multiple internal reflection

The accuracy of Faraday rotation measurements of plane-parallel samples is in many cases determined by the techniques which are used to eliminate or to correct for the effect of the multiple internal reflections on the measured rotation of the plane of polarization.[5-7)] In most measurements of the free-carrier Faraday rotation in the optical and infrared region (except in very thin films[6)]) interference effects[5,6)] are not directly observed. The optical path length is usually much longer than the coherence length $\Delta l\approx\lambda^2/\Delta\lambda$, where λ is the wavelength of radiation. The other coherence condition (a sin u)$\ll$ $\lambda/2$ (where a is the dimension of the emitting element and u the solid angle of the beam), is generally not fulfilled either. Therefore, an average value of transmittance T_a will be measured.[5)] The transmittance for one direction of the magnetic field is given by

$$T_a=\frac{t_1t_2t_1^*t_2^*e^{-\eta d}}{4}\left[\frac{e^{i2(\alpha+\theta)}}{1-Ue^{i4\theta}}+\frac{2}{1-U}+\frac{e^{-i2(\alpha+\theta)}}{1-Ue^{-i4\theta}}\right], \tag{4}$$

where $U=r_1r_2r_1^*r_2^*e^{-2\eta d}$. Intrinsic depolarization is negligible compared with the reflection and transmission terms. Here r_1, r_2 and t_1, t_2 and the Fresnel reflection and transmission coefficients for perpendicular incidence, η is the absorption coefficient, d is the thickness of the sample, α is the angle between polarizer and analyzer, and θ is the Faraday rotation for a single pass. A similar expression for the transmission T_a' can be written for $-\theta$, which corresponds to the reverse field direction.

Two techniques are used to measure the Faraday rotation. In the first one, the polarizer and analyzer are fixed and the rotation is calculated from the measured intensities T_a and T_a'. In the second method, either the polarizer or analyzer is rotated until $T_a=T_a'$; therefore, equal intensities are detected, and the rotation is read directly on the dial of the polarizer or analyzer.

For a fixed angle α between the polarizer and analyzer, and with the intensities corresponding to the two field directions T_a and T_a', the measured rotation γ is generally calculated from the expression

$$\sin 2\gamma=(T_a-T_a')/(T_a+T_a') . \tag{5}$$

For $\alpha=45$ degrees the following expression is obtained from eqs. (4) and (5) for the measured rotation γ as a function of the true rotation θ and the optical constants

$$\sin 2\gamma=\frac{(1-U^2)\sin 2\theta}{1-2U\cos 4\theta+U^2} . \tag{6}$$

For small angles, where $\sin 2\theta\approx 2\theta$, and with the assumption that $\theta\approx\gamma$ in the argument of the harmonic function of the correction term, eq. (6) can be written

$$\gamma\approx\theta(1+2R^2e^{-2\eta d}\cos 4\gamma) . \tag{7}$$

Because of the multiple reflections in the sample, the transmitted beam is also partially depolarized and can be described by the polarization factor ρ.

$$\rho=(T_{\max}-T_{\min})/(T_{\max}+T_{\min}) . \tag{8}$$

From eqs. (4) and (8) one gets

$$\rho=\frac{1-U}{1+U^2-2U\cos 4\theta}[\cos 2(\theta-\gamma) - U\cos^2(\theta+\gamma)] . \tag{9}$$

In the case where there is an observable contribution because of the polar reflection Faraday rotation Φ, the measurement of γ and ρ makes it possible to determine both the rotation in the transmission θ and the polar reflection rotation Φ.[8)]

When the polarizer or the analyzer is rotated until $T_a=T_a'$ (which is usually described as the compensation method, applicable with optical beam splitter methods like the Wollaston prism technique[8)]) the measured rotation is obtained from eq. (4) for this condition by

$$\sin 2(\gamma-\theta)=R^2e^{-2\eta d}\sin 2(\gamma+\theta) , \tag{10}$$

where $R=[(n-1)^2+k^2]/[(n+1)^2+k^2]$. For small rotations, one gets from eq. (10)

$$\gamma\approx\theta+(R^2/2)e^{-2\eta d}\sin 4\theta . \tag{11}$$

For coherent radiation, especially laser radiation, the Faraday rotation exhibits oscillatory behavior.[5)]

The application of the correction for multiple reflections is important for Faraday rotation measurements in semiconductors; for example, one can see from eq. (11) that for rotations of less than 5 degrees and negligible absorption ($e^{-2\eta d}\approx 1$), the measured rotation is 16.4% larger for GaAs and 26% larger for Ge, owing to multiple internal reflections, than the rotation for a single transmission.

§ 3. Experimental Results and Discussion

Faraday rotation measurements on three samples will be described, along with the application of the correction for multiple reflections for incoherent light, as stated above.

Figure 1 shows measurements on n-type GaAs samples. Sample 1 has a carrier concentration

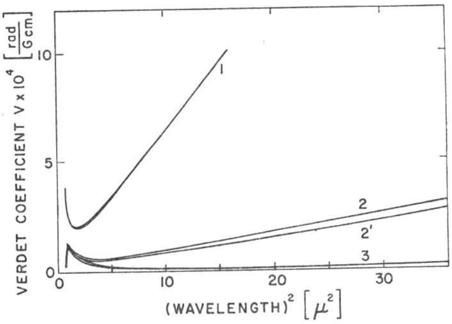

Fig. 1. Free carrier and interband Faraday rotation in n-type GaAs at room temperature and $B=20$ kG. Carrier concentrations N: 1) 7.1×10^{18}; 2,2′) 5×10^{17}; 3) 3.2×10^{16} cm^{-3}. The lower curves (for example, 2′) represent the Faraday rotation corrected for multiple internal reflection effects.

of $N=7.1\times10^{18}\,\text{cm}^{-3}$ and a thickness of $d=$ 100 microns. Sample 2 has a carrier concentration of $N=5\times10^{17}\,\text{cm}^{-3}$, and a thickness of $d=$ 110 microns. Sample 3 has a carrier concentration of $N=3.2\times10^{16}\,\text{cm}^{-3}$, and a thickness of $d=1540$ microns. All samples are tellurium-doped single crystals. Measurements were made at temperatures of 298°K and 77°K at a magnetic field of $B=20$ kG. The figure shows the Verdet coefficient V, which is defined by

$$\theta=VBd\,, \tag{12}$$

as a function of wavelength squared.

Two curves were prepared for each sample. The top curves present the Verdet coefficients calculated from the measured rotation γ. The lower ones (for example, 2′) present the Verdet coefficient as determined by eqs. (7), (11) and (12). The multiple reflection effects are large in Samples 2 and 3. As the absorption coefficient changes with the wavelength, the correction varies between 11 and 15%. The effect is very small on Sample 1, having a correction of about 1.5% in the region between 1 and 2 microns. The effect of low temperature is to increase the Verdet coefficient in the free carrier region between 4 and 8%, depending on the carrier concentration (about 4% for $N=7.1\times10^{18}$. cm^{-3}), in agreement with theory.[13,7] In the interband region close to the absorption edge the Verdet coefficient decreases in degenerate samples. In intrinsic GaAs however, it increases at short wavelengths and decreases at long wavelengths.[9] The following effective masses for the different carrier concentrations have been determined from

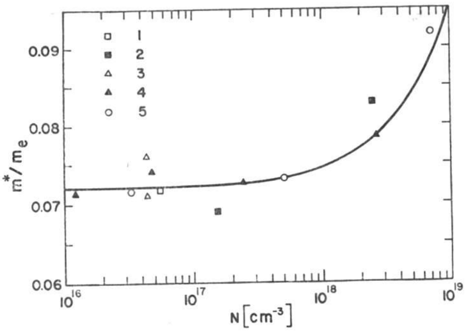

Fig. 2. Electron effective mass m^*/m_e in n-type GaAs as a function of the carrier concentration N in the conduction band. The theoretical curve was calculated for $m_0^*/m_e=0.066$. The points, which were corrected for multiple internal reflection effects, represent the experimental data 1) [12]; 2) [13]; 3) [14]; 4) [15], 5) our results.

the corrected Verdet coefficients using eq. (1): For the carrier concentration of $N=7.1\times10^{18}$ cm^{-3} is $m^*/m_e=0.092$; $N=5\times10^{17}\,\text{cm}^{-3}$, $m^*/m_e=0.073$; $N=3.2\times10^{16}\,\text{cm}^{-3}$, $m^*/m_e=0.071$. The optical constants were taken from published works.[10,11]

Figure 2 shows the experimental effective masses as a function of carrier concentration, in comparison with the theoretical values which were computed from eqs. (2) and (3) for $T=296$°K and $E_g=1.52$ eV.

Other experimental data points shown in Fig. 2, were obtained from various published Faraday rotation measurements.[12–15] The reported effective masses of specimens with low absorption were corrected herewith, for internal multiple reflections using published absorption data.[11,16]

The best agreement of theory with the experimental data is obtained for an effective mass of $m_0^*/m_e=(.066\pm.002)$ at the bottom of the GaAs Γ_{1c} condution band.

The agreement between theory and experiment with respect to the absolute value of (m^*/m_e) and its temperature dependence, seems to indicate that the effect of band population on the measured free-carrier Faraday rotation is relatively small in GaAs. The interband Faraday rotation[9] near the band edge (Fig. 1) is affected significantly by band population effects.[17]

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DISCUSSION

Geist, D.: Your measurements show that the effective mass is constant up to about 10^{18} cm^{-3}. This comes out also by quite different method of measurement, namely static magnetic susceptibility (G. Römelt, D. Geist: Z. angew. Phys. (1962)).

Piller, H.: In the session of band theory M. Cardona reported recent results on the band structure of GaAs determined by using the $k \cdot p$ method. His calculations give the effective mass $m^*/m_e = 0.065$ at $k=0$, which is in very good agreement with our experimental value.

Moss, T. S., Paul, W., Piller, H.: DeMeis and Paul used a wedge shaped sample (1 deg. wedge) which should eliminate the effect of multiple internal reflection on the measurement if the optical equipment is carefully designed to reject all multiple internal reflected light from the detector. Only one transmission of the light beam is considered in the calculation in this case. No other corrections as described in this paper for the case of plane parallel samples would be required. DeMeis and Paul confirmed (ref. 7)) that the effective mass ($m_0^*/m_e^* = 0.064$ at the bottom of the GaAs Γ_{15} conduction band) is low (ref. 15)) compared to the effective mass determined by cyclotron resonance experiments of $m_0^*/m_e = 0.071$. Their experimental values are even smaller than the values determined on plane parallel samples (using the method described in this paper and on wedged samples by Moss and Walton (ref. 12)). For example, the effective mass is 0.071 for a carrier concentration of 1.3×10^{17} cm^{-3} according to Fig. 2. DeMeis and Paul's value for the same carrier concentration was 0.067. This small discrepancy may be explained by including a certain part of the multiple internal reflected polarized light in the calculations of the effective masses from the measured Faraday rotation.