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X-8. Energy Losses by Hot Electrons in Solids: A Semiclassical Approach

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The rates of energy loss by fast electrons to the various types of phonons and electronic excitations are derived in approximate form from an elementary classical model. Energy is emitted as a series of "energy-wells" in the wake of the moving electron. The quantum aspects are introduced as constraints on the classical model.

The rates of energy loss by fast electrons to the various types of phonons and electronic excitations have been analyzed and reported in the literature over the past fifty years. These analyses have for the most part been carried out in momentum space. The following is an attempt to restate the arguments in "real space" and in terms of an elementary model.

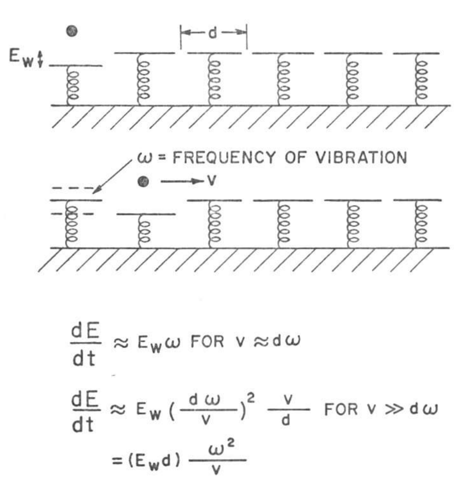

Fig. 1. Model for computing rates of energy loss.

The model in Fig. 1 shows a particle moving with velocity v past a series of elements, each of dimension d. The particle repels each element with a force such that, for a stationary particle, an energy E_w is stored in the compressed spring of the element. Also the frequency of vibration of each element is denoted by ω.

The maximum rate of loss of energy to the array is, by inspection,

$$\left.\frac{dE}{dt}\right|_{\max} \approx E_w \omega \,, \tag{1}$$

and occurs at $v \approx \omega d$. Equation (1), applied to energy loss to polar optical phonons, gives $(\alpha\hbar\omega)\omega$ where $\alpha\hbar\omega$ is the interaction energy of the polaron. This agrees with the results of perturbation theory.[1,2]

The rate of loss of energy, in general, for $v > \omega d$ is

$$\frac{dE}{dt} = E_w \left(\frac{\omega d}{v}\right)^2 \frac{v}{d} = E_w d \frac{\omega^2}{v} \,. \tag{2}$$

The factor $(\omega d/v)^2$ is the reduction in magnitude of the energy-well E_w due to transit of the particle past an element in times short compared with ω. The factor v/d is the number of wells traced out per second.

Equation (2) is the essence of the "real-space" argument. In order to apply it to electrons in a solid the value of the energy-well E_w must be determined in each case. E_w is the interaction energy between the stationary electron and a particular mode of energy loss in the medium.

For those cases where the electron couples to the medium via its coulomb field we can write:

$$E_w = \beta \frac{e^2}{Kd} \,,$$

or, in differential form,

$$\Delta E_w = \beta \frac{e^2}{Kd^2} \Delta d \,,$$

where β is, by definition, the fraction of the available coulomb energy $e^2/(Kd)$ that is used to form an energy-well. K is the dielectric constant for frequencies higher than that of the emitted radiation. Equation (3) is inserted into eq. (2) and integrated to give:

$$\frac{dE}{dt} = \beta \frac{e^2 \omega^2}{Kv} \ln \frac{mv^2}{\hbar\omega} \,, \tag{4a}$$

in those cases where β and ω are constants. The limits of integration are the uncertainty radius $\hbar/mv$ and the adiabatic radius v/ω. In those cases where β or ω are not constant, the maximum

Table I. Rates of energy loss (dE/dt) by electrons of velocity v

Phenomenon	dE/dt	Source
Polar Optical	$\left[\frac{\varepsilon_0-\varepsilon_\infty}{\varepsilon_0}\right]\frac{e^2\omega^2}{\varepsilon_\infty v}\ln\left(\frac{2mv^2}{\hbar\omega}\right)$ Note: $1/2\,mv^2<\hbar\omega$	Frohlich[1)] Callen[2)]
Piezoelectric Phonons	$\frac{\pi}{4}\left[\frac{\varepsilon_p^2}{KC}\right]\frac{e^2\omega^2}{Kv}$ Note: $2\,mv=\hbar\omega/v_s$	Tsu[3)]
Acoustic Phonons	$\frac{1}{4}\left[\frac{B^2\omega^2K}{4\pi e^2\rho v_s^4}\right]\frac{e^2\omega^2}{Kv}$ Note: $2mv=\hbar\omega/v_s$	Seitz[4)] Conwell[5)]
Non-Polar Optical Phonons	$\frac{1}{2}\left[\frac{\pi KD^2}{\rho e^2\omega^2\lambda^2}\right]\frac{e^2\omega^2}{Kv}$ Note: $1/2mv^2>\hbar\omega$ and $\frac{\lambda}{2\pi}=\frac{\hbar}{2mv}$	Conwell[5)]
x-Ray Levels	$\left[\frac{\omega_p^2}{\omega_e^2}\right]\frac{e^2\omega_e^2}{v}\ln\frac{2mv^2}{\hbar\omega_e}$ Note: $\omega_p^2=\frac{4\pi ne^2}{m}<\omega_e^2$ $\hbar\omega_e$=Excitation energy of x-ray levels $1/2mv^2>\hbar\omega_e$	Bohr[6)] Bethe[7)]
Plasma	$[1]\frac{e^2\omega^2}{v}\ln\left(\frac{2mv^2}{\hbar\omega}\right)$ Note: ω=plasma frequency and $1/2mv^2>\hbar\omega$	Bohm and Pines[8)]
Cerenkov	$\left[1-\frac{1}{\varepsilon_\infty}\right]\frac{e^2\omega^2}{v}$ Note: $v=c$=velocity of light in vacuum	See e.g. Schiff[9)]

ε_0 low frequency dielectric constant
ε_∞ high frequency (optical) dielectric constant
K dielectric constant
ε_p piezoelectric constant
C elastic modulus (dynes/cm^2)
ρ density (grams/cm^3)
v_s phase velocity of sound
v velocity of electron
ω angular frequency of radiation
m effective mass of electrons
B deformation potential (electron volts in ergs/unit strain)
D optical deformation potential (electron volts in ergs per centimeter relative shift of sublattices)

contribution to the integral occurs in the neighborhood of a particular radius and becomes:

$$\frac{dE}{dt}\approx\beta\frac{e^2\omega^2}{Kv}. \tag{4b}$$

Equation (4) is the form in which the energy-loss expressions in Table I are recast from the literature. The values of β are set off in square brackets.

It can be shown by an elementary argument that for the losses to phonons β is also equal to the ratio of electrical to total energy of the corresponding macroscopic sound waves. [See RCA Rev. **27** (1966) 96 for calculations of β and their relation to the acoustoelectric effects]. For acoustic and optical phonons coupled by deformation potentials, β was computed as if the slopes of the deformed band edges were a macroscopic electric field. The same result is also obtained directly from eq. (2) by computing the energy well E_w formed by an electron with an uncertainty radius $\hbar/(mv)$.

The value of β for plasmons is unity since the coulomb field of the "stationary" electron is substantially completely cancelled by the polarization field induced in the plasma. Similarly, in the case of the deep-lying or x-ray levels, only the fraction ω_p^2/ω_e^2 of the coulomb field of the electron is cancelled by the induced polarization field. Hence, it is only this fraction of the coulomb energy that is used in forming an energy well.

The quantum constraints on the classical model appear as the threshold conditions that the electron must have at least the energy and momentum of the quanta or radiation it emits. Also, the radius of the electron within which its charge is effectively smeared out is given by the uncertainty relation $r \approx \hbar/(mv)$.

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DISCUSSION

Landsberg, P. T.: Could you say if loss by impact ionization could be included in your last Figure ?

Rose, A.: The application of this model to impact ionization is included in the complete paper due to appear shortly in the RCA Review.