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XI-2. On Amplification of Hypersound and Space Charge Waves in Semiconductors

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Owing to the dependence of the dielectric constant on the deformation of a semiconductor, an additional electron-phonon interaction arises in the case of an externally applied electric field. It may happen to be dominant in crystals with a very large dielectric constant (of the order of thousands). Along with the deformation potential, this interaction is introduced into the theory of the amplification of sound waves by the drift of current carriers. The theoretical value of the amplification factor obtained is not less than in the case of piezoelectric interaction.

A new type of wave is considered, a space charge wave, which can also be amplified at any type of electron-phonon interaction. The introduction of a strong magnetic field raises appreciably the Q-factor of this wave and facilitates the conditions for its realization.

An electron-phonon interaction proportional to the external field is introduced into the theory of mobility. The current as a function of the field will first rise, then fall.

§ 1. Introduction

The possibility of generating hypersound by current carriers was indicated in 1956 by Tolpygo and Uritzky[1] who considered a polaron moving at supersonic velocity. That same year, Weinreich[2] developed independently a detailed theory of the amplification and generation of hypersound by current carriers. In his work, as well as in the more general studies by Spector,[3] Weinreich, Sanders, White,[4] in Kazarinov and Skobov's work[5] and in a number of other studies the electron-phonon interaction was determined by the deformation potential. A stronger electron-phonon interaction and sound amplification take place in piezoelectric crystals in which Hutson, McFee and White were the first to succeed in observing amplification of sound experimentally.[6] The theory of amplification in these crystals was developed by White,[7] Herzenstein and Pustovoit,[8] Gurevich[9] and others.

The present study differs from those mentioned above in the following;

1) A new type of electron-phonon interaction is considered, which is proportional to the externally applied electric field. In crystals having a large dielectric constant of the order of 2000 (such as barium titanate, rutile, SbSJ and the like), this interaction is dominant and would permit to obtain high amplification factors of sound waves if a sufficiently high drift velocity of the current carriers could be produced in such a crystal.

2) A new type of wave (space charge wave) is considered. This wave is amplified when the drift velocity of the carriers is below sound velocity (*i.e.* under conditions opposite to sound amplification). Amplification of the wave can be brought about by any type of electron-phonon interaction. A strong static magnetic field applied to the crystal will appreciably raise the Q-factor of this wave and facilitate its experimental realization.

§ 2. Wave Equations and Their Solution

As we are dealing with waves whose length is greatly in excess of the lattice constant, with the deformations of a crystal and the bunching in it of the concentration of current carriers, the state of the crystal can be described by two independent macroscopic continual degrees of freedom: the vector of elastic displacement of the medium $\boldsymbol{u}(\tau, t)$ and the density of the space charge $\rho(\tau, t)$. The latter is specified by the change in the concentration of electrons not only in the conduction band, but also on those local levels with which the band electrons are able to reach a state of equilibrium at the frequencies being considered. Therefore,

$$e(n-n_0)=q\rho \qquad q\leq 1\,, \tag{1}$$

where e is the charge of the current carrier, n-

their concentration in the conduction band, n_0-the concentration of carriers in a neutral semiconductor, q-frequency-dependent constant taking into account the change in the filling of the local levels mentioned above. In eq. (1) we ignored the comparatively small contribution to ρ due to the change in the concentration of charged donors, caused by the deformation.

Consider, for the sake of simplicity, an isotropic medium. During deformation its dielectric constant changes and becomes a tensor:

$$\varepsilon_{ik}=\varepsilon_0(\delta_{ik}-g_1\delta_{ik}div\, \boldsymbol{u}-g_2u_{ik}), \tag{2}$$

where $u_{ik}=(1/2)(\partial u_i/\partial x_k+\partial u_k/\partial x_i)$ is the tensor of strain, and ε_0, g_1, g_2 are constants. If an average uniform electric field $\boldsymbol{E}_0$ is externally applied to the crystal, then owing to the dependence of ε_{ik} on the coordinates the conduction electron will be acted upon, in addition to $\boldsymbol{E}_0$, by an additional field $\boldsymbol{E}_u$ which is determined from the following equation:

$$(\boldsymbol{\nabla}, \boldsymbol{E}_u)=\left(g_1+\frac{g_2}{2}\right)(\boldsymbol{E}_0, \boldsymbol{\nabla})(\boldsymbol{\nabla}\boldsymbol{u})+\frac{g_2}{2}\boldsymbol{E}_0\Delta\boldsymbol{u}\,. \tag{3}$$

Besides, there is also acting a field $\boldsymbol{E}_\rho$ created by the space charge $(div\boldsymbol{E}_\rho=(4\pi/\varepsilon_0)\rho)$, as well as a force determined by the deformation potential. As a result, the conduction electron is acted upon by the field

$$\boldsymbol{E}^*=\boldsymbol{E}-\frac{b}{e}\boldsymbol{\nabla}(\boldsymbol{\nabla}\boldsymbol{u}) \qquad \boldsymbol{E}=\boldsymbol{E}_0+\boldsymbol{E}_u+\boldsymbol{E}_\rho\,. \tag{4}$$

A unit volume of the crystal lattice is under the action of the force

$$\boldsymbol{F}=(\lambda+\mu)\boldsymbol{\nabla}(\boldsymbol{\nabla}, \boldsymbol{u})+\mu\Delta\boldsymbol{u}+\frac{b}{e}\boldsymbol{\nabla}\rho$$
$$+\frac{\varepsilon_0}{8\pi}\{g_1\boldsymbol{\nabla}E^2+g_2[\boldsymbol{E}(\boldsymbol{\nabla}, \boldsymbol{E})+(\boldsymbol{E}, \boldsymbol{\nabla})\boldsymbol{E}]\}\,. \tag{5}$$

Herein λ and μ are the Lamé coefficients, the first two terms in eq. (5) represent the conventional elastic strain force, the third term is the reaction of the deformation potential force applied to the lattice, b-the deformation potential constant (some average for the band electrons and for the localized electrons which are in equilibrium with them), the last term in eq. (5) denotes a force of the electrostrictive type. A detailed derivation of eqs. (1) to (5) and the elucidation of the meaning of the coefficients q and b are given in eq. (10).

Let us further consider small oscillations of $\boldsymbol{u}(\tau, t)$ and $\rho(\tau, t)$. They are determined by the equations $\gamma\ddot{\boldsymbol{u}}=\boldsymbol{F}$, where γ is the density of the crystal, and $\dot{\rho}+div\boldsymbol{J}=0$. Assuming $\boldsymbol{u}$ and ρ proportional to $e^{i(k\tau-\omega t)}$, we obtain the following equations for the amplitudes of these quantities:

$$(\lambda+\mu)(\boldsymbol{k}, \boldsymbol{u})\boldsymbol{k}+\mu k^2\boldsymbol{u}-\gamma\omega^2\boldsymbol{u}$$
$$=\left[\left(g_1+\frac{g_2}{2}\right)(\boldsymbol{E}_0, \boldsymbol{k})\frac{\boldsymbol{k}}{k^2}+\frac{g_2}{2}\boldsymbol{E}_0+\frac{ib}{e}\boldsymbol{k}\right]\rho\,, \tag{6}$$

$$\left\{\frac{1}{\tau'}+i[q(\boldsymbol{k}, \boldsymbol{v})-\omega]\right\}\rho$$
$$=\frac{\varepsilon_0}{4\pi\tau}\left[\left(g_1+\frac{g_2}{2}\right)(\boldsymbol{k}, \boldsymbol{E}_0)(\boldsymbol{k}, \boldsymbol{u})+\frac{g_2}{2}k^2(\boldsymbol{E}_0, \boldsymbol{u})\right.$$
$$\left.-\frac{ib}{e}k^2(\boldsymbol{k}, \boldsymbol{u})\right]. \tag{7}$$

It is assumed here that in addition to the electric field $\boldsymbol{E}_0$ there exists an external static magnetic field $\boldsymbol{H}$ and the density of the current is equal to

$$\boldsymbol{J}=en\mu_H\boldsymbol{E}^*-eD_H\boldsymbol{\nabla}n \tag{8}$$

where μ_H and D_H are the tensors of mobility and the diffusion coefficient of the current carriers in the presence of a magnetic field. Notations were introduced as follows;

$$\sigma=en_0\mu_H \qquad \frac{1}{\tau}=\frac{4\pi(\boldsymbol{k}, \sigma\boldsymbol{k})}{\varepsilon_0k^2}$$
$$\frac{1}{\tau'}=\frac{1}{\tau}+q(\boldsymbol{k}, D_H\boldsymbol{k}) \qquad \boldsymbol{v}=\mu_H\boldsymbol{E}_0\,, \tag{9}$$

in which $\boldsymbol{v}$ is the drift velocity of the current carriers.

By projecting the vector eq. (6) on a direction perpendicular to $\boldsymbol{k}$ and $\boldsymbol{E}_0$ we obtain the equation of a conventional transverse sound wave. It does not interact with ρ and $\boldsymbol{E}_0$ and is not amplified. The remaining projections of $\boldsymbol{u}$ are determined by projecting eq. (6) on a direction parallel to $\boldsymbol{k}(||)$ and perpendicular to $\boldsymbol{k}$ lying in the plane $\boldsymbol{k}$, $\boldsymbol{E}_0(\perp)$. Introducing the notations

$$s_1=\left(\frac{\lambda+2\mu}{\gamma}\right)^{1/2} \text{—longitudinal sound velocity,}$$

$$s_2=\left(\frac{\mu}{\gamma}\right)^{1/2} \text{—transverse sound velocity,}$$

$$k_1=\frac{\omega}{s_1} \qquad k_2=\frac{\omega}{s_2}$$

$$\mathscr{G}_1=\frac{(g_1+g_2)E_{0\|}+i\frac{b}{e}k}{s_1\sqrt{\gamma}} \qquad \mathscr{G}_2=\frac{g_2E_{0\perp}}{2s_2\sqrt{\gamma}}\,, \tag{10}$$

we obtain from eq. (6)

$$u_\|=\frac{\mathscr{G}_1\rho}{s_1\sqrt{\gamma}(k^2-k_1^2)} \quad u_\perp=\frac{\mathscr{G}_2\rho}{s_2\sqrt{\gamma}(k^2-k_2^2)}\,, \tag{11}$$

and eq. (7) can be written down in the form

$$\left\{\frac{1}{\tau'}+i[q(\boldsymbol{k},\boldsymbol{v})-\omega]\right\}\rho =\frac{\varepsilon_0}{4\pi\tau}\,[\mathscr{G}_1{}^* s_1\sqrt{\gamma}\,u_\parallel+\mathscr{G}_2 s_2\sqrt{\gamma}\,u_\perp]\,. \qquad (12)$$

Equations (11) and (12) determine the wave amplitudes $u_\parallel$, $u_\perp$ and ρ. The solvability condition of these equations is

$$\frac{\varepsilon_0}{4\pi}\frac{\tau'/\tau}{1+i\tau'[q(\boldsymbol{k},\boldsymbol{v})-\omega]}\left\{\frac{|\mathscr{G}_1|^2}{1-\dfrac{k_1{}^2}{k^2}}+\frac{|\mathscr{G}_2|^2}{1-\dfrac{k_2{}^2}{k^2}}\right\}=1\,, \qquad (13)$$

which shows the relation between $\boldsymbol{k}$ and ω, *i.e.* the wave dispersion law. For the given ω and direction of $\boldsymbol{k}$ eq. (13) yields three k-roots. The three corresponding waves are of the mixed charge-acoustical type, since each has all the three quantities $u_\parallel$, $u_\perp$ and ρ different from 0. The roots of (13) are readily determined by tabulation if the parameters ε_0, τ, τ', $\boldsymbol{v}$, q, k_1, k_2, $\mathscr{G}_1$ and $\mathscr{G}_2$ are known.

However, if we assume that the electron-phonon interaction perturbs but slightly the waves which exist in the medium without this interaction, *i.e.* that the right-hand parts of eqs. (6) and (7) are small compared to the separate terms of the left-hand parts, then simple approximate solutions will be obtained for all the three types of waves:

1. For the wave having $k\approx k_1$:

$$\frac{k-k_1}{k_1}=\frac{\varepsilon_0}{8\pi}\frac{\tau'}{\tau}|\mathscr{G}_1|^2\frac{1-i[qv_\parallel/s-1]\omega\tau'}{1+[qv_\parallel/s-1]^2\omega^2\tau'^2}\,, \qquad (14)$$

where $s=\omega/k$ is the phase velocity of the charge-acoustical wave, $v_\parallel$-the projection of $\boldsymbol{v}$ on the direction of $\boldsymbol{k}$. From eq. (11) it follows $u_\parallel\gg u_\perp$, *ie.* this wave is almost longitudinal.

2. For the wave having $k\approx k_2$:

$$\frac{k-k_2}{k_2}=\frac{\varepsilon_0}{8\pi}\frac{\tau'}{\tau}|\mathscr{G}_2|^2\frac{1-i[qv_\parallel/s-1]\omega\tau'}{1+[qv_\parallel/s-1]^2\omega^2\tau'^2}\,. \qquad (15)$$

This wave is almost transverse. In eqs. (14) and (15), k_1 and k_2, respectively, should be substituted for k in the expressions for $\mathscr{G}_1$ and τ'.

3. For the wave having $k\approx k_3\equiv\omega/qv_\parallel+i/qv_\parallel\tau'$:

$$k=\frac{\omega}{qv_\parallel}+\frac{i}{qv_\parallel\tau'}-\frac{i\varepsilon_0}{4\pi\tau qv_\parallel}\left\{\frac{|\mathscr{G}_1|^2}{1-\dfrac{k_1{}^2}{k_3{}^2}}+\frac{|\mathscr{G}_2|^2}{1-\dfrac{k_2{}^2}{k_3{}^2}}\right\}. \qquad (16)$$

For the method of weak electron-phonon coupling used to be valid, the third term in the right-hand part of eq. (16) should be considerably less in modulus than the sum of the first two terms. If waves with rising (in space or in time) amplitude are of interest, it is necessary that the imaginary part of k should be negative, *i.e.* the third term should be greater than the second one in modulus. These conditions lead to the inequality $\omega\tau'\gg 1$ which allows eq. (16) to be written down approximately as follows:

$$k=\frac{\omega}{qv_\parallel}+\frac{i}{qv_\parallel\tau'} -\frac{\varepsilon_0\omega\tau'}{8\pi qv_\parallel\tau}\left\{|\mathscr{G}_1|^2\beta_1{}^2\frac{1-\dfrac{i\omega\tau'}{2}(1-\beta_1{}^2)}{1+\left[\dfrac{\omega\tau'}{2}(1-\beta_1{}^2)\right]^2} +|\mathscr{G}_2|^2\beta_2{}^2\frac{1-\dfrac{i\omega\tau'}{2}(1-\beta_2{}^2)}{1+\left[\dfrac{\omega\tau'}{2}(1-\beta_2{}^2)\right]^2}\right\}, \qquad (17)$$

where

$$\beta_1=\frac{s_1}{qv_\parallel}\,,\qquad \beta_2=\frac{s_2}{qv_\parallel}\,.$$

If β_1 and β_2 are greater than unity, that is, the drift velocity is less than the sound velocity, the corresponding terms will introduce a negative contribution into the imaginary part of k.

The wave under consideration is predominantly a space charge wave, since at the given amplitude ρ in accordance with eq. (11) $u_\parallel$ and $u_\perp$ will be as small as the coefficients of the electron-phonon coupling $\mathscr{G}_1$ and $\mathscr{G}_2$.

The above-mentioned electron-phonon coupling proportional to the applied field is represented in eqs. (6), (7) and (10), eq. (13) by terms containing $\boldsymbol{E}_0$, which enter along with the deformation potential constant b.

§ 3. Discussion of Results. Evaluation of the Amplification Factor. Case of a Strong Magnetic Field

Equations (6)∼(17) differ from those obtained in our study[10] only in that they take into consideration the magnetic field applied to the crystal. Therefore, the mobility and diffusion coefficient are now tensors depending on $\boldsymbol{H}$, and for τ, τ' and $\boldsymbol{v}$ more general expressions are now given and the values of these parameters are also dependent on $\boldsymbol{H}$. For the rest, the formulas have not changed their form.

Inasmuch as Einstein's relation $D_H=(k_0T/e)\mu_H$ is also valid in the presence of a magnetic field, the ratio τ'/τ is independent of H:

$$\frac{\tau'}{\tau}=\frac{1}{1+k^2r^2} \qquad r=\sqrt{\frac{qk_0T\varepsilon_0}{4\pi e^2 n_0}}\,, \tag{18}$$

where k_0 is Boltzman constant, r-Debye shielding radius. In the evaluations below, τ'/τ is assumed to be the order of unity.

The absorption coefficient of two sound waves is equal to

$$\alpha_j=-K_j\frac{\tau'}{\tau}\frac{(qv_{\parallel}/s-1)\omega\tau'}{1+[qv_{\parallel}/s-1)\omega\tau']^2}$$

$$K_j=k_j\frac{\varepsilon_0}{4\pi}|\mathscr{G}_j|^2 \qquad j=1,\,2\,. \tag{19}$$

The maximum value of $-\alpha_j$ is equal to $(1/2)K_j$ and is independent of the magnetic field.

For the evaluation of the coefficient g_1 let us use Lorentz-Lorenz's formula in its generalized form $\varepsilon-1/\varepsilon+2=\text{const.}\times\gamma$. Then we obtain $g_1=(\varepsilon_0+2)(\varepsilon_0-1)/3\varepsilon_0\approx\varepsilon_0/3$ and there is no reason for believing that g_2 might exceed that order of magnitude. If the term with b in the expression for $\mathscr{G}_1$ is neglected in eq. (10), then

$$K_1\cong\frac{\omega\varepsilon_0{}^3E_0{}^2{}_{\parallel}}{36\pi s_1{}^3\gamma}\,. \tag{20}$$

At the frequency 45 megacycles per second, $\varepsilon_0=2000$, $E_{0\parallel}=10^4$ V per cm, $s_1=2\times10^5$ cm per sec, $\gamma=4$ we obtain $K_1=695\text{ cm}^{-1}$. This will correspond to an amplification of sound 1500 db per cm. In the case of a piezoelectric electron-phonon coupling, the amplification factor is also proportional to ω. For the same frequency in CdS the value of the maximum amplification factor computed is equal to 110 db per cm.[6] Thus, the type of electron-phonon coupling as considered in the present study, in substances with an abnormally high dielectric constant, may lead to an amplification of sound no lower than the piezoelectric coupling.

The third wave, the space charge wave, possesses a phase velocity approximately equal to $qv_{\parallel}$ at all the frequencies, and it may be, therefore, called the drift wave. It can be amplified at the expense of the imaginary part of the third term in eq. (17), codsisting of two components each of which as a function of $v_{\parallel}$ has a maximum, respectively, at $qv_{\parallel}=s_j(1-1/\omega\tau')$, $j=1$, 2: $\omega\tau'\gg1$. The half-width of either maximum is equal to $q\Delta v_{\parallel}=2\sqrt{3}\,s_j/\omega\tau'$, *i.e.* it is small. Therefore, the maxima do not overlap and the amplification due to each of the components within the braces of eq. (17) can be considered independently. In the maxima

$$\alpha_j^{\max}=\frac{2}{s_j\tau'}-\frac{1}{2}K_j\,, \tag{21}$$

where K_j is determined by eq. (19). The third wave differs from the first two in that the component $2/s_j\tau'$ appears in the expression for $\alpha^{\max}$. This component is independent of the electron-phonon coupling and causes dampling of the wave. Amplification takes place if the second component predominates in eq. (21), which is also possible in the absence of a magnetic field. Thus, in the numerical example above $E_{0\parallel}=10^4$ V per cm and $qv_{\parallel}\approx s_1=2\times10^5$ cm per sec, hence $q\mu_0=20\text{ cm}^2$ per V$\times$sec, $qD=0.5\text{ cm}^2$ per sec. At the frequency 45 megacycles per second $qDk^2=10^6\text{ sec}^{-1}$. If $n_0=5.3\times10^{13}\text{ cm}^{-3}$, then $1/\tau'=2/\tau=2\times10^6\text{ sec}^{-1}$ and $2/s_1\tau'=20\text{ cm}^{-1}$, which is considerably less than $(1/2)K_1$. However, at higher concentrations of the current carriers the first component in eq. (21) may appear to be predominant.

The inclusion of a strong magnetic field is apt to increase substantially τ and τ', *i.e.* decrease $2/s_j\tau'$ (raise the Q-factor of the third wave). By a strong magnetic field is meant to be the case when the cyclotron frequency of the carrier considerably exceeds the inverse of the time of free path. Then, the tensor μ_H is readily determined from the equation

$$\mu_H\boldsymbol{E}\equiv\mu_0\boldsymbol{e}(\boldsymbol{e},\,\boldsymbol{E})+\frac{C}{H}[\boldsymbol{e},\,\boldsymbol{E}]+\frac{Q}{H^2}[\boldsymbol{E}-\boldsymbol{e}(\boldsymbol{e},\,\boldsymbol{E})]+\cdots$$

$$\boldsymbol{e}\equiv\frac{\boldsymbol{H}}{|\boldsymbol{H}|}\,, \tag{22}$$

where $\boldsymbol{E}$ is an arbitrary vector and μ_0 is the mobility in an isotropic medium in the absence of magnetic field.[11] If the free path time of the current carrier is $t=t_0v^r$, where v is the thermal velocity of the carrier, the constant Q will be determined from

$$Q=\frac{4c^2m}{3\sqrt{\pi}\,et_0}\left(\frac{m}{2k_0T}\right)^{r/2}\Gamma\left(\frac{5-r}{2}\right)$$

$$\Gamma(n)=\int_0^\infty e^{-x}x^{n-1}dx\,, \tag{23}$$

where c is the velocity of light, m-the effective mass of the carrier.

As an example, consider the case of $\boldsymbol{H}\perp\boldsymbol{k}$. Then $(\boldsymbol{k},\,\mu_H\boldsymbol{k})=(Q/H^2)k^2$ is a quantity of the second order of smallness with respect to $1/H$, τ and τ' being large and proportional to H^2. If, in addition $\boldsymbol{H}\perp\boldsymbol{E}_0$, then the drift velocity

$$\boldsymbol{v}=\frac{c}{H}[\boldsymbol{e},\,\boldsymbol{E}_0]+\frac{Q}{H^2}\boldsymbol{E}_0+\cdots \tag{24}$$

and the Joule heat liberation becomes a small quantity $\sim 1/H^2$.

§ 4. Mobility of Current Carriers, Taking into Account the Electron-Phonon Interaction Proportional to the Applied Field

The above-mentioned electron-phonon interaction as the cause of electron scattering was introduced into the kinetic equation along with other scattering mechanisms.[12] The calculation of mobility as based on the kinetic equation has shown that the result differs but 4% to 5% from that obtained from Mattisen's rule according to which

$$\frac{1}{\mu}=\frac{1}{\mu_1}+\frac{1}{\mu_\varepsilon}, \tag{25}$$

where μ_ε is the mobility due only to the above interaction and μ_1-the mobility due to all other scattering mechanisms. For μ_ε the following expression has been obtained[12]:

$$\begin{aligned}\mu_\varepsilon&=\frac{a}{E_0{}^2}\\ a&=6\sqrt{\frac{\pi T_e}{2k_0T^2}}\\ &\cdot\frac{\hbar^2\gamma s_1{}^2}{e(g_1+g_2)^2m^{3/2}(1-\eta)}\left(d_0\ln\frac{d_0+1}{d_0-1}-2\right)\\ \eta&=\left[\frac{g_2s_1}{2(g_1+g_2)s_2}\right]^2 \qquad d_0=\frac{3+\eta}{1-\eta}\end{aligned} \tag{26}$$

T_e-temperature of current carriers.

As a result, the current density will be

$$J=\frac{\sigma_1E_0}{1+\frac{\mu_1}{a}E_0{}^2}, \tag{27}$$

where σ_1 is the conductivity when only the other scattering mechanisms are acting. With growing E_0, J passes through maximum which is achieved at a value of E_0 of the order of 10^5 V per cm. The dropping section of the volt-ampere characteristic is unstable and may be utilized for the generation of oscillations.

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