PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON THE PHYSICS OF SEMICONDUCTORS, KYOTO, 1966
JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN VOL. 21, SUPPLEMENT, 1966

XIX-6. Determination of Fermi Velocity and Effective Mass from Cyclotron Damping of Magneto-Plasma Waves in Bismuth*

H. KAWAMURA, S. NAGATA, S. TAKANO and J. NAKAHARA

Department of Physics, Osaka University, Toyonaka, Osaka, Japan

The magneto-plasma waves in bismuth have been observed in a wide range of magnetic field in the 50 Gc/sec band. For propagation parallel with the bisectrix axis, we have observed a damping in the hole insensitive mode at the neighborhood of the hole cyclotron field. From the positions of the damping edges at both sides, we can determine the Fermi velocity and mass parameters of holes.

§ 1. Introduction

The magneto-plasma waves undergo cyclotron damping[1,2] at the magnetic field given by $\omega_c=\omega+\boldsymbol{k}\cdot\boldsymbol{v}$. For helicon waves in metals ω is much smaller than ω_c, and we can determine the Fermi velocity from this damping edge.[3] However, when the densities of electrons and holes are equal, as in the case of bismuth, we usually observe Alfven waves at the frequency comparable with kv_F, and therefore, ω_c and v_F can't be determined independently from a single damping edge.[2] When the effective masses are anisotropic, we have two Alfven modes with different phase velocities even at a high magnetic field, polarized linearly along the directions of the principal axes of the mass tensor. Although a wave mode, which is cut off at the electron cyclotron field, rotates in electron sense, it was observed to undergo the cyclotron damping in the neighborhood of the hole cyclotron field. From the damping edges of this mode we could determine the Fermi velocity and effective mass of holes.

§ 2. Theoreticals

The dispersion relation for a magneto-plasma wave is obtained from the Maxwell's equations combined with the material equation which connects the current and electric field. If we define the dielectric tensor $\boldsymbol{\varepsilon}(\omega)$ as

$$\boldsymbol{\varepsilon}(\omega)=\varepsilon_0 1-4\pi i\boldsymbol{\sigma}/\omega\ , \tag{1}$$

the wave equation for the electromagnetic field is given by

$$\boldsymbol{k}\times(\boldsymbol{k}\times\boldsymbol{E})+(\omega/c)^2\boldsymbol{\varepsilon}(\omega)\boldsymbol{E}=0\ . \tag{2}$$

The dispersion relation is obtained as the solution of the secular equation for these linear equations. If the nonlocal effect is neglected, the conductivity tensor $\boldsymbol{\sigma}$ can be obtained from the local Boltzmann equation for carriers.

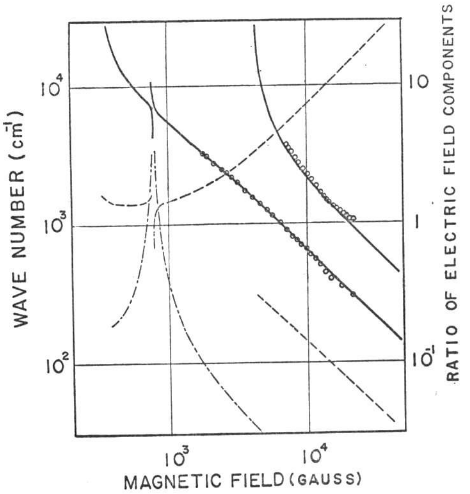
The energy surface for electrons in bismuth is composed of the three tilted ellipsoids.[4] The mass tensor of one of these ellipsoids is defined by m_1, m_2, m_3, and $m_{23}=m_4$, where 1, 2, and 3 are the binary, bisectrix, and trigonal axes respectively. The mass tensors for other ellipsoids are obtainable through rotating this ellipsoid by 120° and 240° about the trigonal axis. The energy surface for holes is a spheroid with longitudinal mass M_l along the trigonal axis and transverse mass M_t. The conductivity tensor under a magnetic field in arbitrary direction can be calculated from these mass tensors.[5] We have computed the dispersion curve, as well as ellipticity E_z/E_x and the longitudinal component ratio E_y/E_x of the electric field for $\boldsymbol{k}//$ bisectrix axis (y-axis) and $\boldsymbol{H}$ in bisectrix-trigonal plane (yz-plane), with the aid of a NEAC-2203 computer, adopting Kao's mass parameters[6] determined from an Azbel-Kaner cyclotron resonance, *i.e.* $m_1=0.0071$, $m_2=1.71$, $m_3=0.0301$, $m_4=0.177$, $M_t=0.0675$, and $M_l=0.76$. In Fig. 1 a few examples for typical orientations are shown. There are two modes, one of which is linearly polarized along x-axis (x-mode) and the other along z-axis (z-mode) at a high magnetic field. The x-mode is elliptically polarized, rotating in hole sense, as the magnetic field is decreased. It is cut off at the hole cyclotron field for the Faraday configuration ($\boldsymbol{k}//\boldsymbol{H}$). The cut off point moves upward as the magnetic field is tilted. The z-mode rotates elliptically in electron sense at lower field having a discontinuity at hybrid resonance field where the longitudinal component becomes very large. It is transparent

* Work supported by Toyo-Rayon Grant.

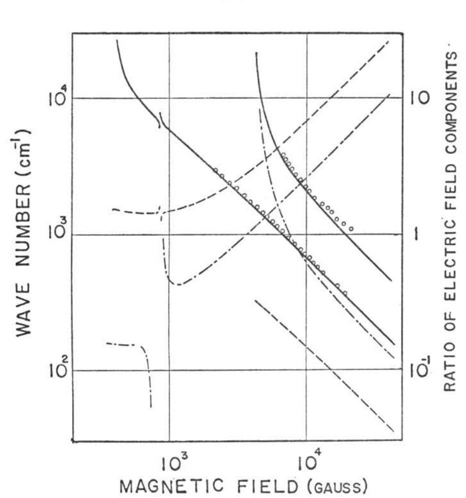
down to the electron cyclotron field.

§ 3. Experimentals and Dispersion Relations

The experiments have been carried out by using a microwave interferometer of 50 Gc/sec at 1.6°K. A specimen of thin slab is inserted between flat faces of two waveguides over holes having a diameter of 3 mm. The ends of waveguides are terminated at $\lambda_g/4$ off the centers of the holes. The microwave transmitted through the specimen is mixed with the by-passed wave.



(a)



(b)

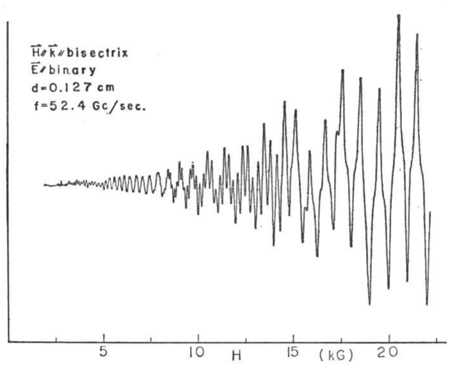
Fig. 1. Wave numbers and wave modes vs. magnetic field.
——; wave number k. -----; E_z/E_x. —·—·—; E_y/E_x. ○; experimental points. Computations were carried out with Kao's mass parameters and carrier density of $2.75\times10^{17}\,cm^{-3}$ for $\omega=2\pi\times52.4$ Gc/sec. $k//y$ and $\boldsymbol{H}$ is in yz-plane.
a) $k//\boldsymbol{H}$. b) $\widehat{k\boldsymbol{H}}=21.5°$.

The specimen was zone-refined for twenty eight times in high vacuum, starting from six-nine bismuth ingot supplied by Asahi Metal Inc. and crystallized in a tapered graphite boat. Both surfaces of the specimen were made plane parallel carefully using spark planer and the surface layers were removed by etching.

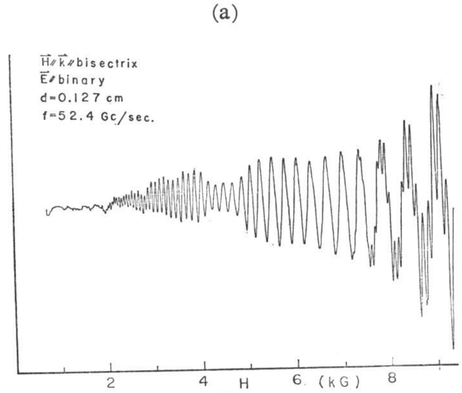
An x-y recorder trace of the signal as a function of magnetic field for the Faraday configuration is shown in Fig. 2. Two series of fringes can be observed in this pattern as expected from the above calculation. The wave number k as a function of H can be obtained from this interference pattern by making use of interference condition $kd=2\pi N$, where d is the thickness of the specimen. In Fig. 1 the white circles are the experimental points obtained by assigning appropriate numbers for N to have the best fit with the theoretical curve in which the carrier density is assumed to be 2.75×10^{17} cm^{-3}.

§ 4. Cyclotron Damping

In Fig. 2 a weak damping region is clearly



(a)



(b)

Fig. 2. xy-recorder trace of interference pattern. $k//\boldsymbol{H}//y$. $\omega=2\pi\times52.4$ Gc. $T=1.6°K$. $d=1.27$ mm. The magnetic field was calibrated with ESR signal of DPPH.

observed in the neighborhood of 4.5 kG which corresponds to the cyclotron resonance field for holes. Although the electric field in z-mode rotates in electron sense, and is insensitive to hole motion in local scheme, it becomes to be hole sensitive, if the nonlocal effect is taken into account. If this damping is due to the excitation of cyclotron motion of holes, the damping edges are given by Doppler shifted cyclotron frequencies $\omega_c+k_1v_F$ and $\omega_c-k_2v_F$, where v_F is the Fermi velocity of holes and k_1 and k_2 are the wave numbers at the lower and upper edges respectively. The positions of damping edges move as shown in Fig. 3.

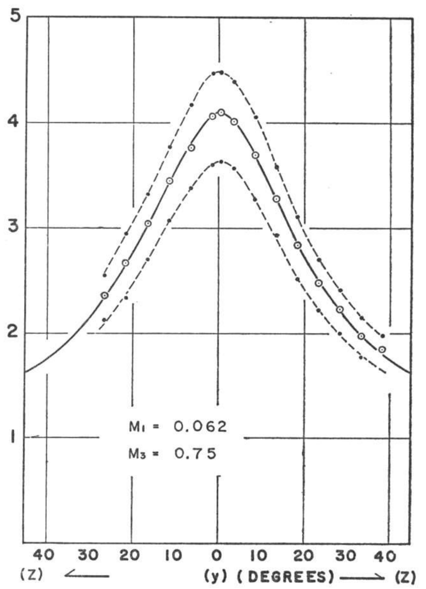


Fig. 3. Damping edges and cyclotron field vs. tilt angle of magnetic field in yz-plane against y-axis. $k//y$ and $\omega=2\pi\times52.4$ Gc. ---●---; experimental damping edge. ○; experimental cyclotron field.

Making use of the values of the wave numbers k_1 and k_2 at the damping edges obtained from Figs. 1 and 2, we can calculate the Fermi velocity and cyclotron resonance field independently from upper and lower damping edges. They are shown in Figs. 3 and 4 for various tilt angles of the magnetic field in yz-plane against the bisectrix axis. In Fig. 4 the theoretical curve for the Fermi velocity of holes calculated from Kao's mass parameters with an assumed hole density of $3.0\times10^{17}\,\mathrm{cm}^{-3}$ is also shown. Agreement with the experiment is reasonable.

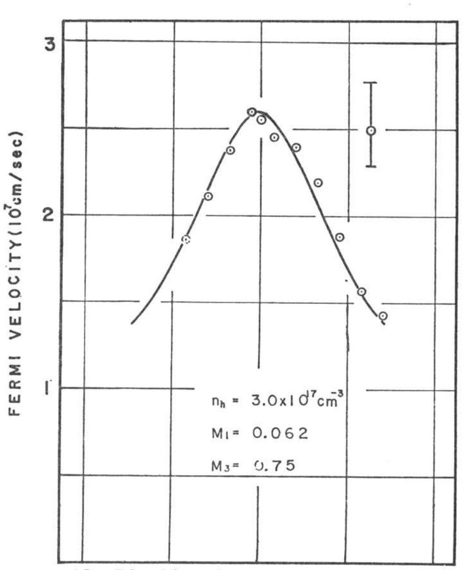


Fig. 4. Fermi velocities of holes in yz-plane. ○; experimental points. ——; calculated curve from present mass parameters.

The mass parameter for holes determined by least square fitting with our experimental values are $M_t=0.062\pm0.002$ and $M_l=0\ 75\pm0.02$, which are compared with the values so far obtained by cyclotron resonance as shown in Table I. The present method of mass determination is quite straightforward and more accurate compared with the old ones.

Table I. Mass parameters for holes in bismuth.

	Kao[8]	Edelman, Khaikin[9]		Present work
		Limiting point	Central orbit	
M_t	0.0675	0.064	0.063	0.062 ± 0.002
M_l	0.76	0.69	0.65	$0.75\ \pm0.02$

References

1) T. Kjeldaas: Phys. Rev. **113** (1959) 1473.
2) J. Kirsch: Phys. Rev. **133** (1964) A1390.
3) M. T. Tailor: Phys. Rev. **137** (1965) A1145.
4) D. Shoenberg: Proc. Roy. Soc. (London) **A170** (1939) 341.
5) B. Lax, K. J. Button, H. J. Zeigers and L. Roth: Phys. Rev. **102** (1956) 715.
6) Y. H. Kao: Phys. Rev. **129** (1963) 1122.
7) V. S. Edelman and M. S. Khaikin: Zh. eksper. teor. Fiz. **49** (1965) 107.

DISCUSSION

Einspruch, N.G.: Did you take any experimental precaution to assure that you were looking at bulk and not surface wave propagation?

Kawamura, H.: No. I did not make any special precaution, but we are certain that the contribution of surface wave was not appreciable, because we used thin specimens. The wave was penetrating through the specimen perpendicularly.