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I.b. Role of Mesons in the Physics of Nuclear Moments

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Magnetic moments caused by meson exchange are discussed. For large separation of two nucleons, the exchange moment is unambiguously predicted by pion theory. In a Fermi gas model it gives $\delta\mu_l \sim 0.1\tau_3$, $\delta\mu_s \sim -0.2\tau_3$ for heavy nuclei and $\delta\mu \sim 0.1$ for the triton. Attempt to deal with the short distance region is presented.

§1. Introduction

First, I would like to ask a question: What is an atomic nucleus made of ? The answer that it is made of protons and neutrons seems obsolete and out-of-date. Instead, I would like to say that a nucleus is made of nucleons, pions and other strongly interacting particles. Effects of pions are often very important and cannot be neglected. Study of nuclear magnetic moments is a good example of where mesic effects are clearly seen. I would like to talk on what pion theory tells us about nuclear magnetic moments. The low energy pion physics is a fairly complete theory, and predictions from it are definite, unambiguous and reliable. The high energy part, however, where many complications can arise, is uncertain and definite results cannot be obtained. Here I will talk about the part for which the pion theory can give a definite prediction, and also mention some attempts to go beyond.

§2. Pion Cloud

In order to get a rough idea, let us consider the behavior of the pion cloud in a nucleus. Consider the simplest nucleus, *i.e.*, proton and neutron. We know that they are not point-like elementary particles: they have structure. First, they have anomalous magnetic moments in addition to normal Dirac moments. Second, their charge and magnetic moment are distributed over an extended region. The spatial extension of this region can be measured by high-energy electron-nucleon scattering and the result is that the extension is of the order of 0.8 fm. More exactly, charge and magnetic moment density is given by an exponential function and the root mean square radius is equal to 0.8 fm.

These facts show that a nucleon is in fact made of a nucleon and pions. A nucleon is composed of a bare nucleon and a charged pion cloud circulating around it (Fig. 1). For the wave function this is a "configuration mixing": a physical proton is a superposition of a bare proton state and the state of a bare neutron and a positive pion of various momenta.

$$\psi_p = \psi_p + \sum_k \psi_{n\pi}(k) \ .$$

In Feynman graphs, a nucleon spends a part of time emitting pions and reabsorbs it to return to its original state. The average pion momentum $\sim 1/0.8$ fm $= 2\kappa$, κ being the pion mass.

When this nucleon is brought into a nucleus, the situation is shown in Fig. 2. The average

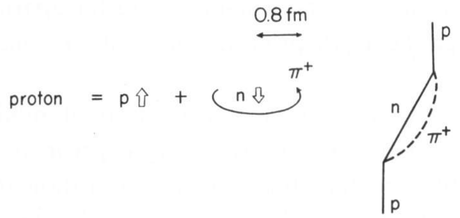

Fig. 1.

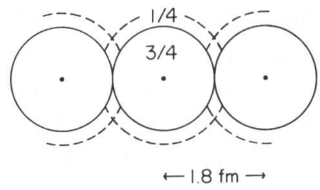

Fig. 2.

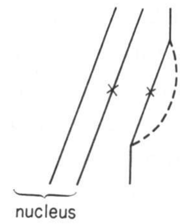

Fig. 3.

internucleon distance is $\sim$ 1.8 fm and the pion cloud cannot remain as it was since it overlaps with the other clouds. The tail of the cloud, which constitutes about a quarter of the anomalous magnetic moment, must be affected in some way.

The situation is easier to understand in momentum space. In a nucleus a nucleon is not free to move: some states are already occupied and transitions to such states are forbidden by the Pauli exclusion principle. Part of the transitions to form a pion cloud is excluded if the recoil nucleon has momentum less than the Fermi momentum P_F(Fig. 3). So a fraction of the pion cloud corresponding to $k<P_F$ is cut out by this effect. This should result in a quenching of the anomalous magnetic moment inside a nucleus.

The fact that the anomalous magnetic moment changes in a nucleus reminds us of dielectrics in which the Maxwell equations are modified and the effective charge of a particle changes because of the dielectric constant. I propose to name nuclear matter "dimesics". In dimesics the pion dynamics is modified from those in vacuum, so that masses, anomalous moments, nuclear forces, ets., take different value from those in free space. I hope Rho will talk on this subject another day.

§3. Exchange Magnetic Moments

The dimesic effect due to the exclusion principle is equivalent to pion exchange effect among nucleons. If one forgets the exclusion principle in the intermediate states, the process of Fig. 4(a) is no longer forbidden, so no change of this effect. However, the process Fig. 4(b) is also allowed, which gives equal contribution as the exclusion of Fig. 4(a). In calculating the dimesic effect, one can choose either way; exclusion principle or two-body exchange effect. Here we take adopt the two-body approach.

A two-nucleon system has an extra exchange magnetic moment in addition to the sum of

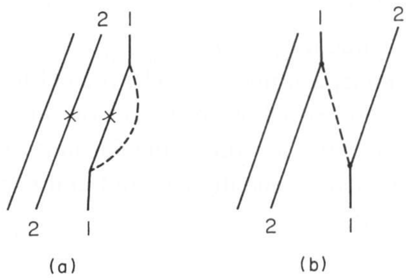

Fig. 4.

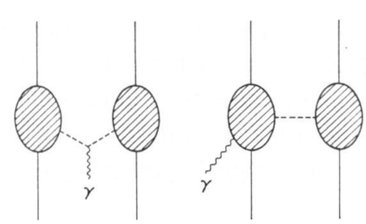

Fig. 5.

each moment. We investigate this operator from outside, *i.e.*, first when they are far apart. In this case only the lightest particle can travel between them, which means that only the one pion exchange process takes place.

We calculate matrix elements shown in Fig. 5. This is connected with the photopion production process which is very well investigated. Comparison with low energy experiments has shown that the static approximation is applicable and that the isovector transition is dominant over isoscalar transition, that is, the photon is absorbed by a nucleon in the form $\tau\sigma$ and not σ. In this approximation, the exchange magnetic moment can be expressed exactly in terms of the known pion-nucleon scattering amplitude.

This expression has been obtained by Kuroboshi and Hara.[1] Their result is

$$\mu(x) = -\frac{e}{2}(\tau^1 \times \tau^2)_3\left[\left\{\frac{r(\sigma^1 \times \sigma^2\cdot r)}{r^2}\left(1 + \frac{1}{\kappa r}\right) - \sigma^1 \times \sigma^2\right\}\frac{f^2}{4\pi\kappa}e^{-\chi r} - (x^1 \times x^2)V(x)\right]$$
$$+ [-\tau_3^1\nabla(\sigma^1\nabla) + (\tau^1 \times \tau^2)_3(\sigma^1 \times \nabla)(\sigma^2\nabla)f(r) + (1 \leftrightarrow 2)],$$
$$f(r) = \frac{2}{9\pi}(\mu_p - \mu_n)\int\frac{\sin^2\delta_{33}}{k\sqrt{k^2+\kappa^2}}dk\frac{e^{-\kappa r}}{r} = 0.033\frac{e^{-\kappa r}}{\kappa r}(nm) \tag{1}$$

f is the pion-nucleon coupling constant, δ_{33} is the pion-nucleon scattering phase shift in the state $j = 3/2$ and $i = 3/2$, and V is the one pion exchange nuclear potential. The first two lines represent the Born approximation which Prof. Brown called one-pion-exchange term. To the one-pion-exchange term I include the third line of eq. (1) which is a rescattering correction.

The one pion exchange magnetic moment (1) is the correct one in the outside region $r \gtrsim 1/\kappa$ and must be included in calculating nuclear magnetic moments.

§4. Pion Exchange Contribution to Nuclear Moments

In order to see how big is the contribution of exchange moments, we have calculated the dimesic effect, *i.e.*, the change of magnetic moment in nuclear matter. We sum particle 2 over all nucleons in nuclear matter, which we take to be a Fermi gas, to obtain the effective magnetic moment of particle 1. The expression becomes simple in momentum space and the change of the magnetic moment in dimesics is written as

$$\delta\mu = \tau_3\left\{\delta\mu_s\sigma + \delta\mu_l l + \delta\mu_t\left(\frac{p\sigma p}{p^2} - \frac{\sigma}{3}\right)\right\} \tag{2}$$

where

$$\delta\mu_s = -0.17, \quad \delta\mu_l = 0.10 \quad \delta\mu_t = 0.13 \ . \tag{3}$$

These values depend on momentum p^2 and (3) is their value for $p = P_F$.

$\delta\mu_s$ is the quenching of the anomalous magnetic moment in nucleus. The second term is proportional to l, which means that the orbital g-factor is enhanced by 10%. The origin of this change can be understood in the following way. Suppose that a proton of angular momentum l makes a virtual transition into a neutron state of angular momentum m and emits π^+ of angular momentum $l - m$. Magnetic moment of this state is

$$\frac{e}{2E_\pi}(l - m) \ .$$

In dimesics part of the intermediate states are excluded and results in the change

$$\delta\mu \propto \sum_{m\varepsilon \text{ nucleus}} \frac{e}{2E_\pi}(l - m) \ .$$

Since there are as many positive m as negative m, the m term vanishes and the change $\delta\mu$ is proportional to l.

If one looks at the Schmidt diagram (Fig. 6), the general trend is a large quenching of $\delta\mu_s$ and a slight enhancement of $\delta\mu_l$. Of course $\delta\mu$ is not due to pions only. Deformed core states also contribute to the effective $\delta\mu$. Generally, this contribution is negative both for $\delta\mu_s$ and $\delta\mu_l$, so, positive $\delta\mu_l$ means that effect of pions is essential in this case. We shall hear in this conference how $\delta\mu_l$ and $\delta\mu_s$ are measured and compared with theory.

For light nuclei such as ^{3}H or ^{3}He, the Fermi gas model is not applicable and we have to calculate separately. With a simple wave function, we get

$$\delta\mu(^3\mathrm{H}) = 0.13$$

which is of correct sign and right order of magnitude. Actually, $\delta\mu_s$ depends sensitively on the nuclear wave function. r dependence of the exchange moment contains a term as shown in Fig. 7. For the triton the positive part dominates while for a Fermi gas of high density, positive and negative contributions tend to cancel each other. With the two-nucleon correlation, $\delta\mu_s$ can be positive or negative and large. So $\delta\mu_s$ can be used as a test of the wave function. $\delta\mu_l$, on the other hand, is rather insensitive to the choice of the wave function.

Incidentally, the non-Born term of the exchange magnetic moment (1) can also be used for calculating the change of the Gamow-Teller coupling constant G_A by a simple replacement $(\mu_p - \mu_n)/2 \rightarrow G_A$.

In the above discussion we took the standard way, namely, we have eliminated pion states and transformed them into a static exchange moment. However, since pions are as important as other states, I would like to require configuration mixturers to mix pion states in addition

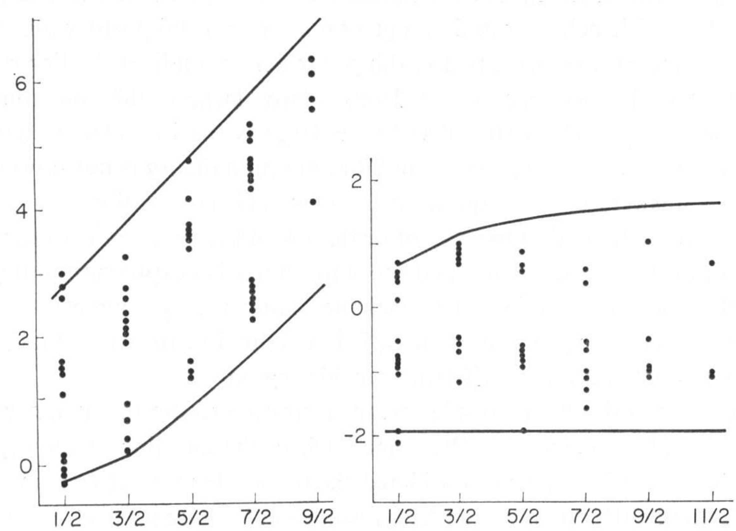

Fig. 6.

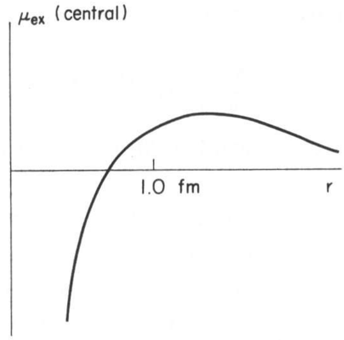

Fig. 7.

to the polarized core states in their wave function. Certainly pionic states have large energies but the interaction matrix elements are also large. We have more information on the pion-nucleon interaction than on the nucleon-nucleon interaction.

Is it possible to see how much of the magnetic moment is due to the nucleon and how much is due to pions? There is no way of clearly distinguishing these two, but the following can be mentioned. The pionic contribution is of isovector type, *i.e.*, of equal and opposite sign for protons and neutrons. The nucleon current has both isovector and isoscalar components of about equal magnitude. The Schmidt lines are very asymmetric for odd proton and odd neutron nuclei, but deviations from Schmidt lines are of almost purely isovector character. This can be easily understood if a considerable amount is of pionic origin. Of course the possibility that all deviations are due to nucleon currents is not excluded if the isoscalar deviation is shown to be small.

§5. Inside Region

The exchange moment in the outside region $r > 1/\kappa$ is given by the one pion exchange part (1). It has a favorable behavior in that it gives positive $\delta\mu_l$ and positive $\delta\mu_s$ for light nuclei.

On the other hand, in the inside region things are very complicated. For $r < 1/\kappa$, two or more pions can be exchanged between nucleons. How large is the contribution of heavy mesons ? How large is the relativistic effect? How large is the isoscalar exchange moment ? How about the three body exchange moment ? Present pion theory is not powerful enough to give unambiguous answers to these questions. We would like to discuss in this conference how these effects can be treated. One way of dealing with the problem is to use the one-pion-exchange moment in the outside region and use some empirical expression in the inside region as is done for the nuclear potential. The exchange moment, as compared with the nuclear potential, is less singular at the origin, that is, it is weighted against the small r region. It is expected that main contribution comes from outside region.

Here, as one way of discussing inside region, I propose to use the quark model to go beyond the one-pion-exchange process. We come back to the question at the beginning: What is a nucleus made of? So far we have considered the model that a nucleus is made of nucleons and one pion. Actually, the one-pion-model is insufficient because it fails to explain anomalous moments of both proton and neutron: it gives too large isoscalar anomalous moment. On

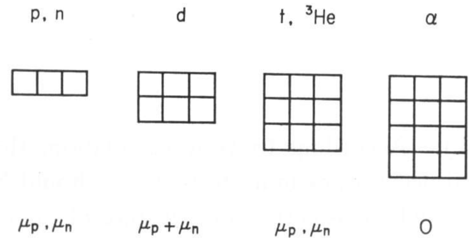

Fig. 8.

the other hand, both the proton and neutron moment can be explained if one assume that a nucleon is made of three quarks. Quarks are spin 1/2 particle of charge 2/3 and $-1/3$ and up to three quarks one can take a symmetric configuration. With this model, $\mu_p : \mu_n = 3 : -2$ in perfect agreement with observation. So, at least as far as the magnetic moment is concerned, it is more reasonable to consider a nucleon as made of quarks rather than of a bare nucleon and a pion.

The quark wave function has an extension of radius 0.8 fm. In the outside region everything is dominated by one pion exchange. However, if two nucleons come closer to $r < 1.6$ fm $\approx 1/\kappa$, two quark wave functions begin to overlap and give rise to an extra moment. This moment is proportional to X^2, X being the overlap of wave functions. This exchange moment is regular at the origin and is a smooth function of r. So, if the one-pion-exchange moment is continued to this function at $r = 1/\kappa$, this corresponds to a cutoff for small r and gives small correction to the one-pion-exchange value obtained above.

However, the adiabatic approximation which fixes the nucleon centers and considers quark exchange, is not a good one for heavy quark. Instead, we should consider a nucleus as made of $3A$ quarks, where A is the mass number. For light nuclei the wave functions in the simplest form are shown in Fig. 8. The α-particle forms a closed shell and up to this all quark can take the same S-orbit. In this simple configuration magnetic moments are given by the sum of nucleons and no exchange correction. This will mean that only a small correction comes from the inside region.

This model is too simple: it gives G_A/G_V ratio of $5/3 = 1.67$ as compared with the observed value 1.2. Some other states such as $4q + \bar{q}$ state must be mixed to the nucleon wave function and this will fix both the magnetic moment and the Gamow-Teller constant of a nucleon. Such mixture is also expected to give corrections to $\mu(d)$, and $\mu(t)$.

Quark Fermi gas model gives $\delta\mu$ which depends on the coupling scheme of orbital and spin angular momentum.

In conclusion I would emphasize that the one-pion-exchange moment (1) in the outside region is a definite prediction from the pion theory and must be included in the calculation of nuclear moments. In a future nuclear theory, we would like to consider a nucleus made of nucleons and pions of made of quarks.

Reference

1) E. Kuroboshi and Y. Hara: Progr. theor. Phys. **20** (1958) 163, errata **21** (1959) 768.

Discussion

A. ARIMA (Stony Brook): According to your calculation, the mesic effect on $\delta\mu_s$ is -0.17n.m. However, the Ml-decay rates indicate that $\delta\mu_s$ should be of the order of 1 n.m. Thus, the mesic effect is too small. This is the best evidence of the importance of core polarization. I now have a question: how large a modification of $\tau\sigma$ do you expect from the mesic effect?

MIYAZAWA: I expect the modification of the Gamow-Teller interaction, from meson theory, to be zero for the triton and about a 15% reduction for a Fermi gas. Is that favorable to you?

ARIMA: From the core polarization point of view the correction is also negative, so if I have both of them we could have too large a modification. Now I'm asking another question, namely, how large is the higher order contribution such as a tensor force exciting a nucleus followed by another meson exchange?

MIYAZAWA: I have classified such an effect as an inside effect, a two pion exchange. I'm not sure how to calculate it, since the tensor force is very singular. By the way, what I call the rescattering correction is almost an isobar effect. This vanishes for the triton as Prof. Brown has mentioned. But it makes the contribution for $\delta\mu_s$ equal to about -0.3 for heavy nuclei, so I would say that the isobar effect is very important in heavy nuclei.

E. KANKELEIT (Darmstadt): For studies of the hyperfine anomaly it would be important to know if the mesic correction could be different, maybe appreciably different, at the outside of a nucleus from the correction on the inside. One should expect this, at least from the Feynman graph of your talk.

MIYAZAWA: I have considered nuclear matter which is infinite. For a finite sized nucleus there may be some change from the values which I have given for the case of an infinite nucleus. I must calculate the exchange moment with a finite sized nuclear wave function.

G. BROWN (Stony Brook and Nordita): It should be clear that there is substantial agreement between the talks of Professor Miyazawa and myself. The one thing that is different is that in our work we consistently obtain a positive δg_s from mesonic corrections. This results because of the short range repulsion from the strong interactions, which tends to make the mesonic corrections to δg_s positive.

MIYAZAWA: The exchange moment is positive outside and this makes the triton moment lie outside the Schmidt line. For a Fermi gas of high density, the negative contribution from inside tends to cancel the positive contribution, but we still get a positive $\delta\mu_s$ from the Born term. However, from the isobar term, we get, a very big quenching which makes the whole thing negative. So the point is whether this isobar effect is big in a correct nuclear model.