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II.f. Nuclear Polarizabilities and Static Quadrupole Moments of Light Nuclei

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In Coulomb excitation the polarization of a nucleus in the electric field of the partner in the scattering process leads to a reduction of the cross section for E2 excitation. First experimental evidence for this polarization effect in the projectile excitation of ^{6,7}Li is presented. Its implication for reorientation experiments in light nuclei is discussed.

§1. Introduction

In a not too strong external electric field, $\boldsymbol{E}$, the nucleus assumes an electric dipole moment, $\boldsymbol{d} = \alpha\boldsymbol{E}$. This dipole moment can be viewed as an admixture of states of the giant dipole resonance in the nuclear wavefunction and the polarizability, α, is thus related to total E1 photonuclear cross sections. The interaction energy of the induced dipole moment with the electric field implies an energy shift $\Delta V = -\alpha\boldsymbol{E}^2/2$. In Coulomb excitation this energy change is time-dependent and leads to a modification of the cross section for E2 excitation in much the same way as the time-dependent hyperfine interaction of the electric field gradient and the nuclear quadrupole moment leads to the reorientation effect.[1,2] The study of the E1 effect is partly undertaken to determine its influence on the analysis of reorientation experiments.

A quantitative estimate of the change in the probability for E2 excitation from the ground state $|\mathrm{i}\rangle$ to an excited state $|\mathrm{f}\rangle$ can be made in second order perturbation theory[2] which yields

$$\frac{P_{\mathrm{if}}^{(\mathrm{E1})}}{P_{\mathrm{if}}^{(\mathrm{E2})}} = -118.2\frac{E}{Z}\phi(\xi\,\theta)\frac{S(\mathrm{E1})}{\langle\|\mathcal{M}(\mathrm{E2})\|\mathrm{f}\rangle}\,. \tag{1}$$

In this expression Z is the charge number of the nucleus being excited, E is the bombarding energy in the center of mass system (in MeV), $\langle\mathrm{i}\|\mathcal{M}(\mathrm{E2})\|\mathrm{f}\rangle$ is the E2 matrix element (in e fm^2), $\phi(\xi\theta)$ is a ratio of orbital integrals for E1 and E2 excitation,[2] and

$$S(\mathrm{E1}) = \sum_n W(11\,I_\mathrm{i}I_\mathrm{f};\,2I_n)\frac{\langle i\|\mathcal{M}(\mathrm{E1})\|n\rangle\langle n\|\mathcal{M}(\mathrm{E1})\|\mathrm{f}\rangle}{E_n - E_i} \tag{2}$$

is in e^2 fm^2/MeV. The sum is over all intermediate states $|n\rangle$ which are connected by E1 excitation with both ground state and excited state. In even-even nuclei only $J^\pi = 1^-$ states contribute and $S(\mathrm{E1})$ can be related in the framework of the hydrodynamic model to the nuclear polarizability yielding[2-4]

$$\frac{P_{\mathrm{if}}^{(\mathrm{E1})}}{P_{\mathrm{if}}^{(\mathrm{E2})}} = -0.02\frac{AE}{Z^2}\phi(\xi\theta)k_0\,, \tag{1.a}$$

where A is the mass number and k_0 is related to the (-2) moment of the photoabsorption cross section, $\sigma_{-2} = \int \sigma(E) E^{-2} dE$, by[5)]

$$\alpha = 2e^2 \int \frac{|\langle n|Z|i\rangle|^2}{E_n - E_i} = \frac{\hbar c}{2\pi^2} \sigma_{-2} = \frac{\hbar c}{2\pi^2} k_0 A^{5/3} (\mu\mathrm{b/MeV}) . \quad (3)$$

It was shown by Winther[3,4)] that the E1 polarization effect can be conveniently incorporated in existing computer codes for evaluating Coulomb excitation cross sections by replacing the usual quadrupole interaction $V^{(2)}$ with an effective one

$$V^{(2)}_{\mathrm{eff}} = V^{(2)} (1 - qa/r(t)) . \quad (4)$$

Here a is half the distance of closest approach, $r(t)$ is the projectile-target distance and $q = 9.6\, E\, S(\mathrm{E1})/(Z\langle i\|\mathscr{M}(\mathrm{E2})\|f\rangle)$ (or $q = 0.0016\, k_0 AE/Z^2$ in the hydrodynamic model).

It is evident from eqs. (1) and (4) that large E1 polarization effects are obtainable for projectile excitation of light nuclei where they may dominate the deviations of the cross sections from those predicted in first order perturbation theory. Figure 1 shows the relative magnitude of processes contributing significantly to projectile excitation of 22 MeV ^{7}Li. The E1 contribution can be isolated experimentally in three different ways:

a) by a decrease of the cross section at all angles, requiring independent knowledge of relevant E2 matrix elements

b) by its angular dependence which is much weaker than that of the reorientation effect

c) by its linear increase with bombarding energy.

§2. Experiments in ^{7}Li and ^{6}Li

Conclusive evidence on the E1 polarization effect has come from projectile excitation of the 0.478 MeV, 1/2$^-$ state in ^{7}Li in experiments at Heidelberg[6)] and Chalk River.[7)] Accurate excitation cross sections were obtained from direct particle spectra using thin targets,[6)] or by a particle-gamma coincidence technique.[7)] In the latter work particles were detected at $\sim 173°$ in an annular detector and at 90° in a telescope consisting of 33 μm and 2000 μm thick detectors. Gamma-rays were detected in five 12.7 cm diam × 15.2 cm NaI (Tl) detectors of which the photopeak efficiencies were measured with radioactive sources. Spectra of gamma rays, particles, and their time relationship observed in coincidence at the lowest bombarding

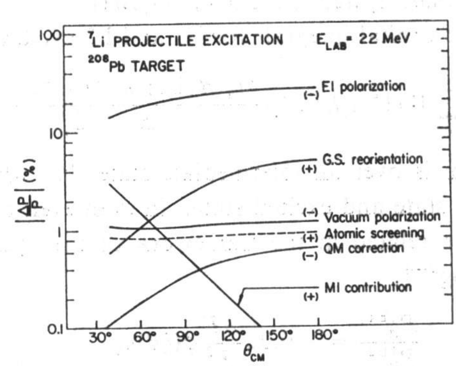

Fig. 1. Angular dependence of various contributions to Coulomb excitation of the 1/2$^-$, 0.478 MeV state in ^{7}Li.

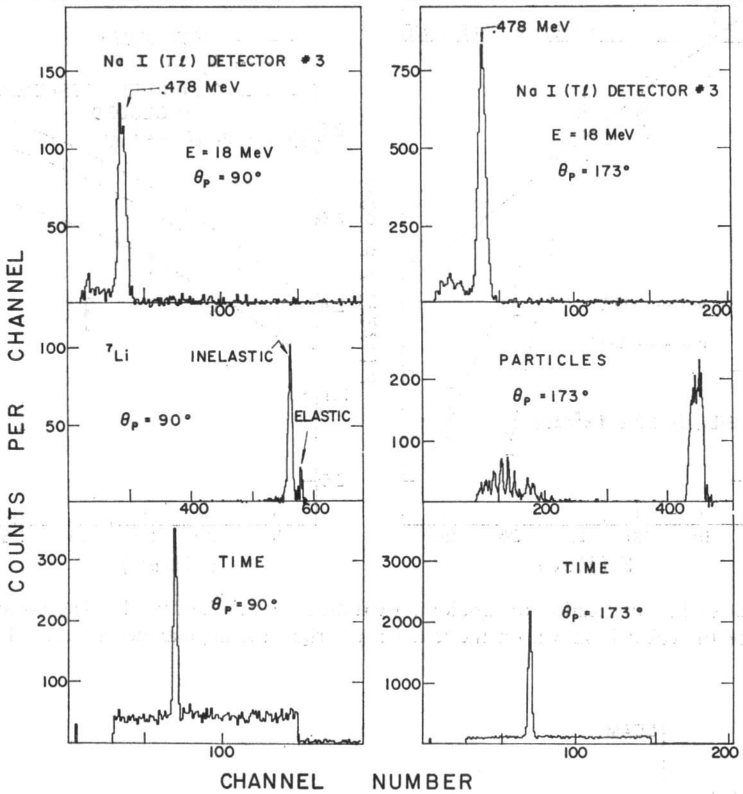

Fig. 2. Energy spectra of gamma rays, particles and their time relationship, observed in coincidence at $\theta_p = 90°$ and $173°$.

energy, are shown in Fig. 2. Singles spectra were recorded concurrently to allow normalization to Rutherford cross sections. The results are shown in Fig. 3. The data show a clear deviation from calculations which ignore the E1 polarization. The present analysis includes the effects of ground-state reorientation ($Q(3/2^-) = -3.66 \pm 0.03$ e fm^2 from ref. 8, quantum-mechanical corrections, and contributions from M1 excitation. The energy dependence of the cross sections at 90° and 173° and the ratio of cross sections at energies below 24 MeV can be consistently fitted with $S(\mathrm{E1}) = 0.028 \pm 0.004$ e^2 fm^2/MeV. The Heidelberg data[6] were subjected to the identical analysis and yielded a somewhat smaller $S(\mathrm{E1})$ which agrees within errors with the quoted result.

In ^{6}Li the $B(\mathrm{E2}, 1^+ \rightarrow 3^+)$ is accurately known from inelastic electron scattering[9] and radiative capture experiments.[10] The E1 polarization effect can thus be estimated from the reduction of the cross section.[11] The experiment is complicated because of the breakup of the 2.18 MeV, 3^+ state ($\Gamma = 25$ keV) into $\alpha + \mathrm{d}$. The lifetime of the state is, however, considerably longer than the collision time in Coulomb excitation and a sequential process is thus expected.

In a kinematically complete experiment deuterons and α particles were detected in coincidence in two separate particle counter telescopes. The experimental arrangement is shown

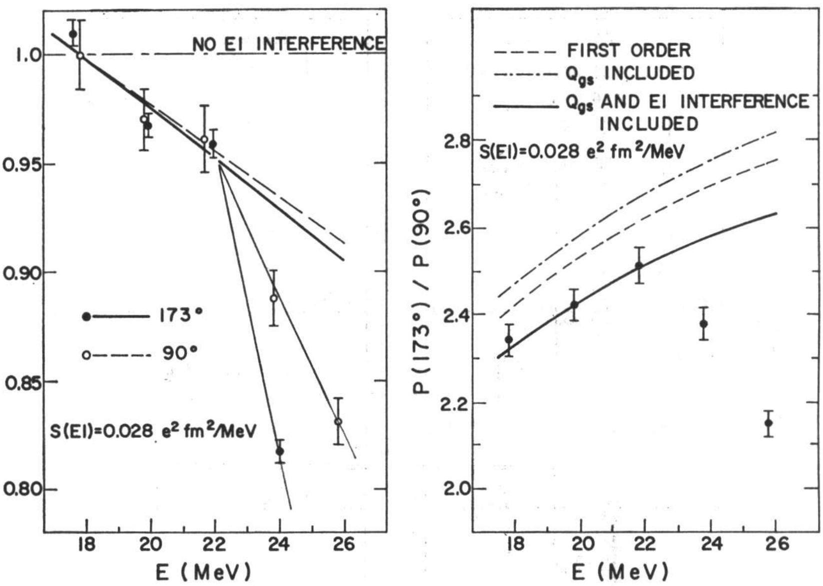

Fig. 3. Effect of E1 polarization on Coulomb excitation probabilities in ^{7}Li. The linear energy dependence of the effect is shown on the left. On the right the angular dependence of the effect is shown.

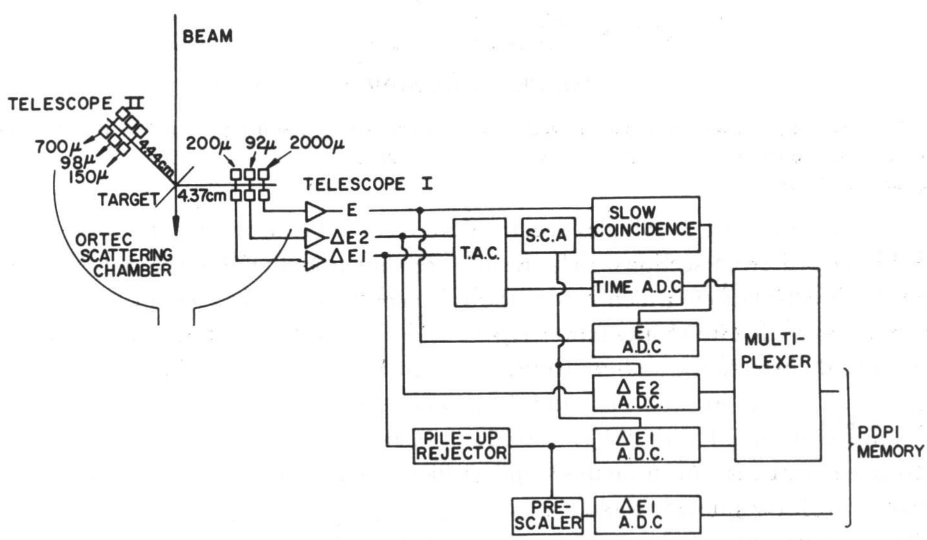

Fig. 4. Experimental arrangement used to study the Coulomb breakup of ^{6}Li.

schematically in Fig. 4. The ΔE1 detector was thick enough to stop scattered ^{6}Li and α particles emitted near 180° in the ^{6}Li test frame, the longer range deuterons emitted near 0° were identified in ΔE_2–E detectors. A two-dimensional spectrum of the deuteron and α energies and a spectrum summed along a kinematic line is shown in Fig. 5. The single strong peak near $E_{\mathrm{d}} = 10.8$ MeV can be identified with Coulomb breakup of the 2.18 MeV state.

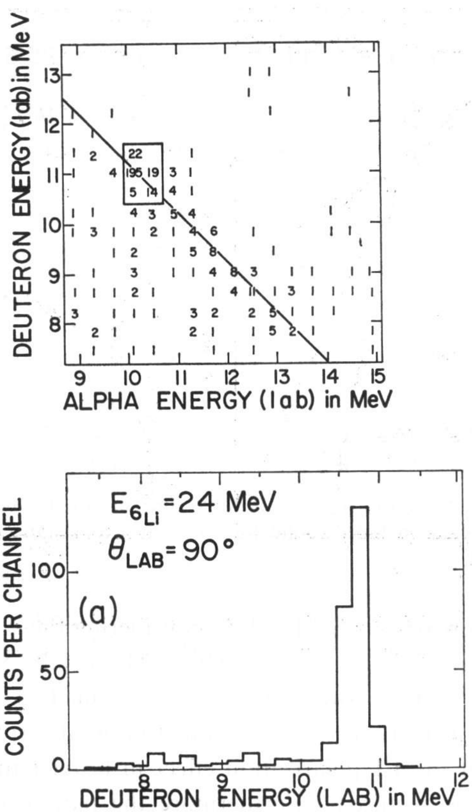

Fig. 5. Upper half: Two dimensional spectra of α and deuteron energies observed at $E_{\text{Li}} = 24$ MeV. Lower half: deuteron spectrum obtained after summation along the kinematic line shown in the upper spectrum.

The results on excitation probabilities obtained after efficiency and angular distribution corrections, are shown in Fig. 6. Only a few data points were obtained because of the low coincidence efficiencies of the telescopes ($3\text{–}5 \cdot 10^{-5}$). The solid line in Fig. 6 takes the reorientation effect in the 1^+ and 3^+ states into account and $S(\text{E1}) = 0.035\ \text{e}^2\ \text{fm}^2/\text{MeV}$ was assumed.

§3. Discussion

The quoted values of $S(\text{E1})$ for ^{6,7}Li are in agreement with the predictions of the hydrodynamic model (*e.g.* (1.a)) if experimental photoabsorption data[12–14)] are used to evaluate σ_{-2}. This is surprising because such a simple model is not applicable to light nuclei and, furthermore, not all states with a certain J that are reached from the ground state by E1 excitation (0^-, 1^-, 2^- for ^{6}Li) are connected to the excited state (only 2^- for ^{6}Li). In ^{6}Li a schematic calculation has been performed[15)] to estimate $S(\text{E1})$. All states were treated in the LS-coupling limit and only excitations 1p → (2sld) were considered. A similar model[16)] explained very well the observed ^{6}Li(γ, n) cross sections. Radial integrals were calculated with a Woods-Saxon potential and the resulting $S(\text{E1}) = 0.034\ \text{e}^2\ \text{fm}^2/\text{MeV}$ is in good agreement with the value assumed in Fig. 6.

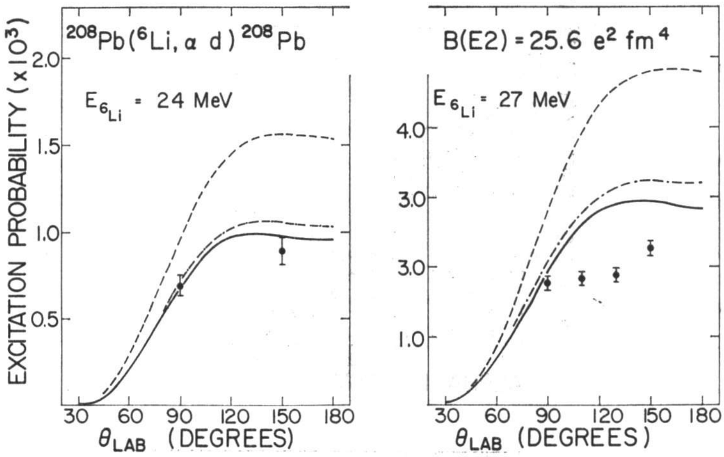

Fig. 6. Probabilities for excitation of the 2.18 MeV state in ^{6}Li. Dashed line: first order estimate. Dot-dashed line: E1 effect included. Solid line: E1 effect and static quadrupole moments of 1^+ and 3^+ states included.

In ^{7}Li S(E1) was estimated by Smilansky[6] considering the α + t channel only and using scattering phaseshifts and radiative capture data. The result, $S(\mathrm{E1}) \sim 0.014$ e^2 fm^2/MeV, is about half the observed value. Although the α + t channel has the lowest threshold energy it contributes only a small fraction to the integrated photoabsorption cross section.[14]

The measurement of the E1 polarization effect in heavier nuclei is feasible in principle although high accuracy is required. Furthermore the parameter k_0 decreases for heavier nuclei as is shown in Fig. 7. It has recently been pointed out by Ericson and Hüfner[17] that the polarizabilities may be observed more directly from transitions in exotic atoms (K^- $\overline{P}$ Σ^- etc). Sizeable energy shifts up to 50 eV are produced by the sum of nuclear and hadron polarizabilities and should be observable by experiment.

§4. E1 Polarization and Reorientation Experiments in Light Nuclei

A number of reorientation experiments have been performed in recent years (cf. ref. 18) and the E1 polarization effect has frequently been ignored in their analysis. Unfortunately the data are not sufficiently accurate or plentiful to deduce the E1 effect directly. Inclusion of the E1 effect in the analysis tends to increase extracted B(E2) and also Q_{2^+}. The changes are largest for the Berkeley experiments.[19] Using the hydrodynamic estimate (eq. 1(a)) the B(E2) values increase by up to 6%, while the increase in $Q_{2^+} \lesssim 0.02$ eb. Both changes are within the quoted experimental errors.

A summary of presently known quadrupole moments in (sd)-shell nuclei[18] is shown in Fig. 8. The experimental values are compared to the rigid rotor model (solid lines), to a large shell model calculation,[19] and a projected Hartree-Fock(HF) calculation.[20] The agreement of experiment with the HF results of Lee and Cusson is remarkable (with the exception of ^{32}S which exhibits a vibrational energy spectrum) considering that no effective charge was required. The quadrupole moments $|Q_{2^+}|$ of ^{20}Ne, ^{22}Ne, and perhaps ^{24}Mg and ^{18}O, seem to be larger than the transition moments even after taking the E1 effect into account.

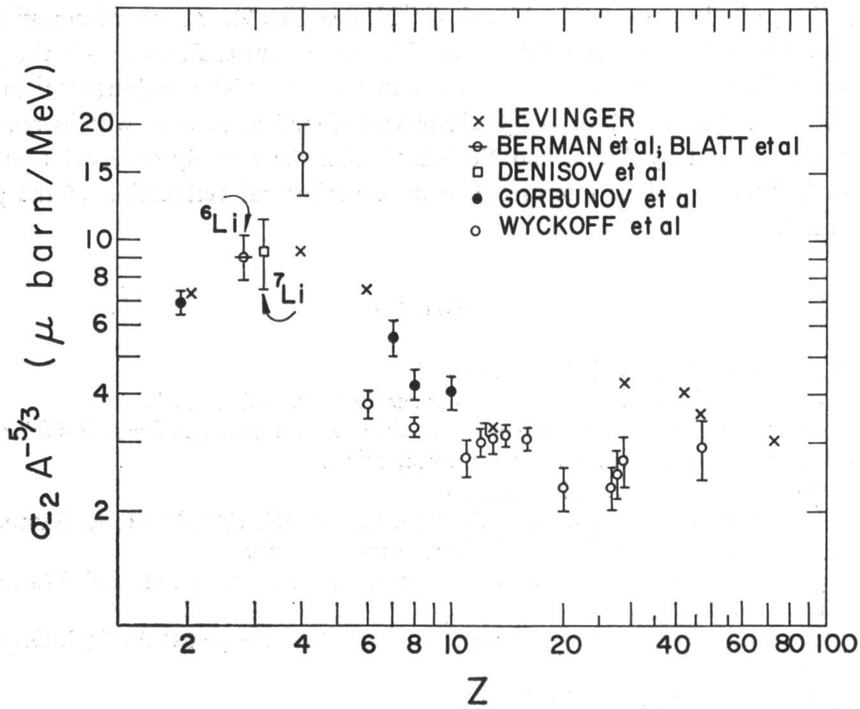

Fig. 7. The (−2) moment of total photoexcitation cross sections for various nuclei.

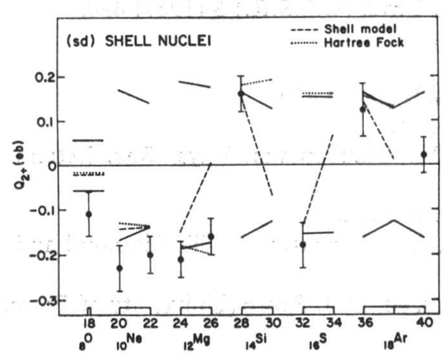

Fig. 8. Static quadrupole moments of 2^+ states in nuclei of the (sd) shell. The solid lines are the rigid rotor prediction.

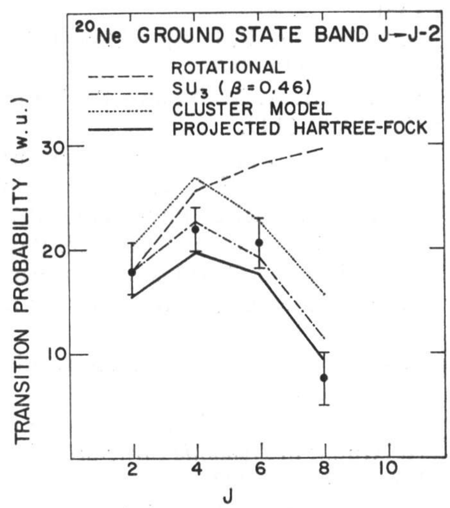

Fig. 9. E2 transition probabilities in ^{20}Ne compared to several theoretical predictions.

This deviation from the simple rotational model is not expected in the HF model which otherwise predicts for these nuclei a reduction of the deformation of the intrinsic state as the spin in the ground state band increases. This prediction is in accordance with recent measurements[21] in the ground state rotational band of ^{20}Ne (see Fig. 9). Another interesting feature of the HF Calculation is the predicted decrease of the rms radius as J increases in the g.s. band. A qualitatively similar effect has been obtained in cluster-model calculations[22] in ^{20}Ne in which the average distance of the α – ^{16}O clusters was found to decrease with increasing spin. It has been pointed out by Arima and Yoshida[23] that the recently observed trend[24] in α decay width in ^{20}Ne could be interpreted as an experimental verification of the predicted changes in radii.

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Discussion

A. R. Barnett (Manchester Univ.): As an alternative to the perturbation theory treatment you describe, one can include the giant dipole resonance directly into the Winther-deBoer calculation. The price one pays is, of course, a great increase in computing time, but one does avoid the renormalization of the quadrupole operator you describe and retains the coupled channels aspect of the program. We did this when the Manchester group measured the quadrupole moments of $^{20,22}Ne$ and found a 3% effect, similar to the values that you quote. In the case of ^{20}Ne we have good information, from (p,γ) work on ^{19}F, on the matrix elements for transitions up to the giant resonance and down to the 2_1^+ state, so that the calculated correction is reasonably well supported by experiment.

A second comment: Professor Arima referred to a possible discrepancy in ^{20}Ne (and other cases) where the 2^+ quadrupole moment seems to be larger than the rotational-model value deduced from the $B(E2)$. In the Manchester measurement we point out that one trouble with this comparison is that usually the experiments are not designed to obtain both $B(E2)$ and $Q(2^+)$. We were able to do this in a consistent way and found a $B(E2)$ somewhat greater than (but in agreement with) the DSAM/Chalk River results as well as a $Q(2^+)$ somewhat smaller than (but also in agreement with) the Berkeley and Heidelberg results. Both these trends tend to reduce the possible, discrepancy and our results are $Q_0=-0.176$ and $Q(2^+)=-0.20\pm 0.066$. We feel this discrepancy probably does not exist.

HÄUSSER: There are a number of effects which tend to reduce the discrepancy between the transition moment and quadurpole moment in ^{20}Ne. Apart from the E1 effect there is a small contribution due to the large hexadecapole deformation of ^{20}Ne which is in the right direction. It is my feeling that the discrepancy is likely to disappear after more extensive and thorough projectile reorientation experiments are performed.

E. KANKELEIT (Darmstadt): Data on nuclear polarization in an external electric field are already available from muonic atoms. These results are in reasonable agreement with calculations by Chen and Skardhammer.

HÄUSSER: In exotic atoms one may obtain polarizabilities even in light nuclei by going to lower orbits. Of course the hadron polarizability also may contribute to the energy shift.