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V.h. Nuclear Moments and Related Structure of Vibrational and Transitional Nuclei

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(Presented by T. Tamura)

It is shown that the Boson expansion technique describes quite well the transition from vibrational ^{148}Sm to deformed ^{154}Sm. It is also shown that it predicts large quadrupole moment of the 2_1^+ state of ^{114}Cd, without making the $2_2^+ \rightarrow 0_g^+$ transition probability too large.

To explain the properties of the so called vibrational nuclei has been one of the pending problems of the theoretical nuclear physics in the past two decades, and in fact many attempts have been made with different degrees of success.[1] However, recently our qualitative understanding of these nuclei has been modified significantly, motivated by two major types of experiment. One is the discovery of the large static quadrupole moment, Q_2, of the first excited 2^+ state in these nuclei,[2] while the other is the measurement of the energies of the yrast 0^+, 2^+, 4^+, 6^+ . . . states, which showed that $\Delta E_I = E_I - E_{I-2}$ continue to increase with increasing I.[3] Both of these experiments show that the vibrational nuclei, believed to be basically spherical, have features that resemble rather strongly those of deformed nuclei.

This fact does not of course mean however that a theory which successfully describes deformed nuclei can also explain the vibrational nuclei. Rather it means that one should attempt to construct a theory which can describe vibrational and deformed nuclei, and consequently transitional nuclei, on one footing, rather than to construct a theory which is specifically attempting to explain only the vibrational nuclei. In this regard it is worthwhile to note two recent works reported by Bes, Greiner and their co-workers[4] who showed that, if a Hamiltonian for an quadratic oscillater is supplemented with appropriate unharmonic terms and if this whole Hamiltonian is diagonalized in a large multiphonon space, it is possible to obtain theoretical spectrum that have a nature of that of well-deformed nuclei. Since the same theory of course gives purely vibrational spectrum, if the unharmonicity term is suppressed, we see that a theory which we were seeking for has been found. Only task remains is to derive an appropriate Hamiltonian, the derivation being made in some microscopic way.

Since it seemed to us that the Boson expansion technique was one of the best ways, known to date, to serve this purpose we started to work on it some two years ago. The basic idea of this technique was first proposed by Belyaev and Zelevinsky,[5] and later was extended and applied by Sørensen[6] to a fairly large number of nuclei. In many respects our approach[7] follows very closely that of Sorensen. However, we have succeeded in solving the equations for the Boson expansion coefficients exactly. This has been done so far to sixth order and to continue it further, if necessary, does not offer any unsurmountable difficulty. Because of this

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and of the introduction of a few additional algebraic techniques, the Boson Hamiltonian has been obtained in an extremely compact form, including terms that are of sixth order in the collective branch of the Bosons. In the numerical calculations which has been done so far and are to be presented here, however, only terms up to the fourth order have been included, although the coupling of the collective branch with the noncollective branches was taken into account in an approximate way, using a specific form of the closure approximation. It turned out that this coupling played a rather significant role, in getting good agreement with experiment. The effective interaction taken was a sum of particle-hole type quadrupole-quadrupole interaction and the pairing interaction of both monopole and quadrupole nature. The strength parameters of these three forces will be denoted as χ, G_0 and G_2, respectively.

In order to see to what extent we can describe the transition of the nuclear properties from vibrational to deformed, by going through the periodic table, we first chose Sm isotopes in which it is known that ^{148}Sm has a strong vibrational nature, while ^{154}Sm is considered as one of the most typical deformed nucleus, ^{150}Sm and ^{152}Sm then having rather typical transitional nature. Actually a similar calculation has already been made by Sørensen,[6] and he succeeded in reproducing to a large extent the different properties of ^{148}Sm and ^{152}Sm. In his calculation, however, the absolute magnitudes of the excitation energies, particularly of ^{152}Sm, were about a factor two too large compared with experiment, perhaps because his multiphonon space was not large enough, and also because the coupling between the collective and noncollective branches was not taken into account sufficiently. These two difficulties do not exist in our calculation, and we in fact obtained satisfactory agreement of the energy spectra of the yrast states for all these four isotopes ^{148}Sm- ^{154}Sm, including the absolute values of the excitation energies. This can be seen in Fig. 1.

In order to show further that the electromagnetic properties of these nuclei can also be described rather well by our theory, we compare in Table I the experimental and theoretical values of Q_2 and $B(\mathrm{E2}; 2_1^+ \to 0_g^+)$. As is seen, the agreement is quite good though a few small discrepancies do exist. In this table theoretical values of Kumar[8] are also given, which agree with experiment to more or less the same degree as do our results. Note, however, that in fitting the data of ^{150}Sm and ^{152}Sm, Kumar introduced an adjustable parameter, the "inertial renormalization factor," and thus his treatment of the kinetic energy part of the Hamiltonian has been phenomenological to some extent. Also no calculation has been made for ^{148}Sm.

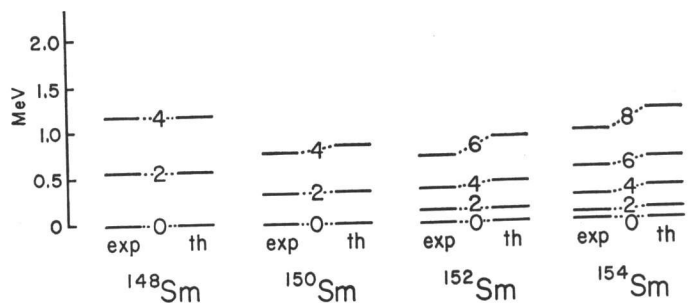

Fig. 1. Comparison of theoretical and experimental energies of states of the bound band in Sm isotopes.

Table I. Q_2 and $B(E2; 2_1^+ \rightarrow 0_g^+)$ in Sm isotopes. Theoretical values in parentheses are those of Kumar (ref. 9).

	A	148	150	152	154
Q_2	exp	-0.97 ± 0.27[a]	-1.31 ± 0.19[a]	-1.65 ± 0.19[b]	
(eb)	th	-0.96	$-1.12(-0.95)$	$-1.56(-1.64)$	$-1.85(-1.97)$
B(E2)	exp	0.151 ± 0.01[c]	0.272 ± 0.010[d]	0.670 ± 0.015[d]	0.843 ± 0.019[d]
(e^2b^2)	th	0.187	0.276(0.232)	0.595(0.650)	0.841(0.942)

a) D. Cline *et al.*, presented at this conference V–18.
b) G. Casper *et al.*: *Proc. Conf. on Heavy Ions, Heidelberg 1969.*
c) R. M. Diamond *et al.*: Phys. Rev. **C3** (1971) 344.
d) F. S. Stephens *et al.*: Phys. Rev. Letters **29** (1971) 1151.

In Fig. 2 we compare the experimental and theoretical spectra of ^{150}Sm and ^{152}Sm, including states that belong to bands other than the ground band. As is seen, the theoretical heads of the β-and γ-bands appear too high, although the spacing between states that belong to each band are of right order of magnitude. This will mean that the potential energy surface also drawn in Fig. 2 gives the equilibrium value of β_2 correctly, but is too steep. We expect that the sixth order terms, that have not been included so far, will make this potential energy surface less steep, and bring down the band heads. In Table II a few B(E2) values between states that appeared in Fig. 2 are presented, and as is seen they agree with experiment rather well, although fairly large discrepancies are occurring sometimes. We expect that the sixth order terms will again remove these difficulties. Table II also gives theoretical values of Kumar.[8] They agree with experiments somewhat better than do our present results.

Having thus seen that our theory works rather well for Sm isotopes, we shall now turn to

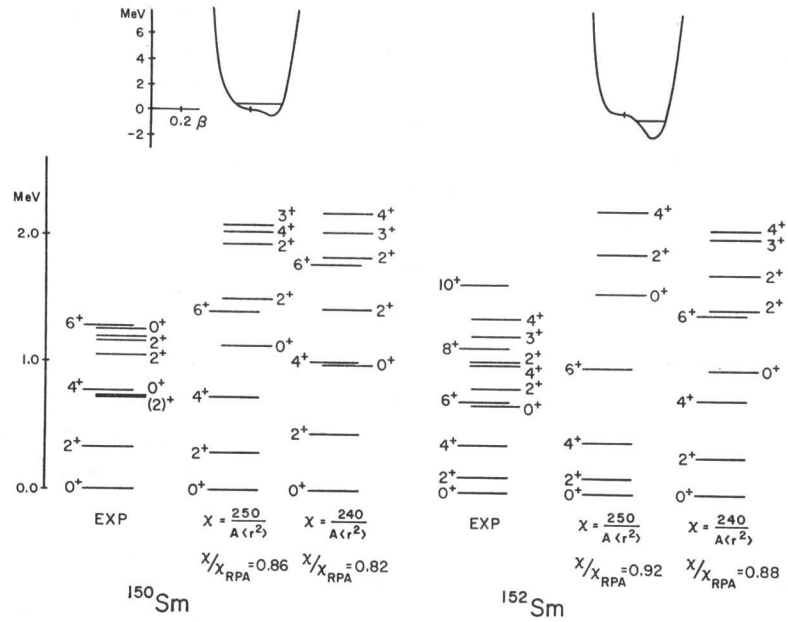

Fig. 2. Comparison of theoretical and experimental spectra for ^{150}Sm and ^{152}Sm.

Table II. Comparison of B(E2) values in ^{150}Sm and ^{152}Sm. Units are e^2b^2, and the theoretical values in parentheses are those of Kumar (ref. 9).

Transitions	^{150}Sm exp	^{150}Sm th	^{152}Sm exp[c]	^{152}Sm th
$2_g \to 4_g$	0.94 $\pm$ 0.08[a]	0.86(0.77)	1.81 $\pm$ 0.06	1.55(1.79)
$0_g \to 2_\beta$			0.023 $\pm$ 0.002	0.032(0.016)
$4_g \to 2_\beta$			0.051 $\pm$ 0.006	0.017(0.076)
$0_g \to 2_\gamma$	0.048 $\pm$ 0.010[b]	0.100(0.048)	0.090 $\pm$ 0.012	0.085(0.109)
$2_\gamma \to 0_\beta$			0.0007 $\pm$ 0.0007	0.001(0.0008)
$2_\gamma \to 2_\beta$			0.28 $\pm$ 0.04	0.013(0.250)

a) R. M. Diamond *et al.*: Phys. Rev. **C3** (1971) 344.
b) R. J. Keddy *et al.*: Nuclear Phys. **A113** (1968) 676.
c) R. G. Stokstad *et al.*: Phys. Rev. Letters **27** (1971) 748.

^{114}Cd, which has been considered as one of the most typical vibrational nuclei, and to explain its large Q_2 together with other related quantities has been one of our major concerns. As far as Q_2 is concerned, Tamura and Udagawa[9] proposed sometime ago a model in which the 2_1^+ and 2_2^+ states are written, respectively, as $a|1;2> + \sqrt{1-a^2}|2;2>$ and $a|2;2> \sqrt{1-a^2}|1;2>$, where $|N;I>$ stands for a purely vibrational state with N phonons and a spin I. When the mixing coefficient a is determined so that the ratio $B(E2; 2_2^+ \to 2_1^+)$ over $B(E2; 2_1^+ \to 0_g^+)$ fits the experiment, the calculated $|Q_2|$ is about 0.3 eb, in good agreement with experimental value $-(0.3 \sim 0.4)$ eb. The difficulty of this naive modes was that the ratio $R = B(E2; 2_2^+ \sim 0_g^+)/B(E2; 2_2^+ \to 2_1^+) = 0.14$ predicted by this model was about one order of magnitude too large compared with experimental value $R = 0.015$, although a similar but a more microscopic approach reduced R to 0.047.

In our present calculation, it turned out that the major part of the 2_1^+ state wave function has much the same nature as it was in our old model, thus giving $Q_2 = -0.37$ eb, in good agreement with experiment.[10] The amplitude of the $|1;2>$ state in 2_2^+ state turns out to be about the same again as in our old model. However, we now have other components in 2_2^+ state. Also 0_g^+ state has mixture of higher phonon states. Thus the $2_2^+ \to 0_g^+$ transition amplitude consists of several terms of a similar magnitude and they cancel with each other, giving rise to $R = 0.0003$, which is now about 50 times too small. This smallness however does not need to be taken as a serious difficulty. We have so far been treating only the collective branch explicitly, which might become a poor approximation in the vibrational region. We believe that the situation is improved largely, if our calculation is modified so that a few noncollective branches are also treated explicitly. A trouble possibly of a similar origin can be seen in Fig. 3, in which the theoretical spectrum of ^{114}Cd is compared with experiment. As is seen the energies of 0_2^+ and 4_1^+ states are too large. It is expected, however, that the noncollective branches explicitly introduced will press down these states and thus remove this difficulty, although the sixth order term may also play an important role here too.

Figure 3 also gives spectra of ^{106}Pd and ^{122}Te. Since ^{106}Pd behaves more or less similarly as does ^{114}Cd, we shall not comment about it any further. As for ^{122}Te, the potential energy surface given in Fig. 3 indicates that this nucleus favors an oblate doformation giving $Q_2 > 0$, in contradiction with experiment. We found, however, that it is possible to make $Q_2 < 0$, by

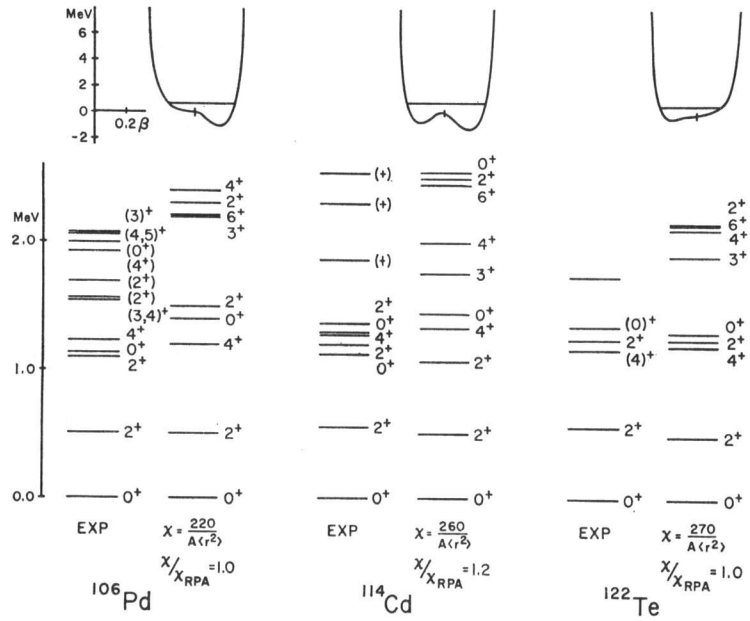

Fig. 3. Comparison of theoretical and experimental spectra for ^{106}Pd, ^{114}Cd and ^{122}Te.

slightly increasing the strength parameter G_2 beyond the value $G_2 = \chi/4$, which has been used in almost all the calculations reported in the present article. Generally speaking the theoretical results for vibrational nuclei seem to be more sensitive, than are those for deformed nuclei, to the choice of various parameters, like single-particle energies, force constants and so forth, and thus will require somewhat more extensive search of such parameters.

Summarizing, we may say that our approach works quite well in explaining many important features of various collective nuclei, in spite of the fact that calculations so far made are of preliminary nature. Admittedly, our calculations still have difficulties in various aspects, but we have also seen that there seems to exist ways to remove them. Thus, for example, if we are in or close to the deformed region, the inclusion of the sixth order term will help to improve the situation. On the other hand, if we are in or close to the vibrational region, the explicit treatment of the noncollective branches are expected to improve the theory. Such extended calculations are under preparation, and results will be obtained shortly.

References

1) References to previous works can be found in refs. 7 and 9 below.
2) J. deBoer and J. Eichler: Advances in Nuclear Physics **1** (1968) 1.
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8) K. Kumar: Nuclear Phys. **92** (1967) 653; Phys. Rev. Letters **26** (1971) 269; Chapter III in *The Electromagnetic Interaction*, ed. W. D. Hamilton (North Holland Publishing Company), to be published.
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Discussion

CHAIRMAN: Because of the close connection between this paper by Dr. Tamura and the following one by Dr. Sørensen, we would like to proceed to next talk and then after that we shall have discussion on both of these papers.