

## V.j. Magnetic Moments and Electromagnetic Transition Rates of the Anomalous Coupling States with Spin $I = j - 1$

A. KURIYAMA, T. MARUMORI<sup>†</sup> and K. MATSUYANAGI<sup>††</sup>

*Department of Physics, Kyushu University, Fukuoka*

<sup>†</sup>*Institute for Nuclear Study, Tokyo University, Tanashi, Tokyo*

<sup>††</sup>*Department of Physics, Kyoto University, Kyoto*

(Presented by K. Matsuyanagi)

Various electromagnetic properties of the anomalous coupling states (ACS) with spin  $I = j - 1$  are shown to be well explained by a new point of view on structure of the ACS. In the new point of view, the ACS are considered as the dressed three-quasiparticle modes which are regarded as a kind of elementary excitation modes in odd mass nuclei.

§1. The anomalous coupling states (ACS) with spin  $I = j - 1$  have been known as the typical phenomena in which the conventional phonon-quasiparticle-coupling theory<sup>1)</sup> is in complete breakdown.

The importance of unharmonic effects or deviations from the random phase approximation (RPA) have already become clear in even-even nuclei. It suggests to us the existence of some hidden correlations in the "unharmonic effects," which may be difficult to take into account properly *within the "phonon space."* In clarifying the hidden correlations and to establish a new microscopic model, the importance of an odd-mass system has come to be recognized, since the existence of the odd-quasiparticle may reveal the hidden correlations in a clear way through their interplay with "phonons."

In recent years, a number of new examples of the ACS with spin  $9/2^-$  have also been found in Cd, Te and Xe isotopes close in energy to the single-quasiparticle  $11/2^-$  state. Furthermore, the measurement on the electromagnetic properties, such as  $B(E2)$ ,  $g$ ,  $B(M1)$ , have been providing us important information, showing various aspects of the structure of ACS.<sup>2-9)</sup> The characteristics of the electromagnetic properties of the ACS with spin  $j - 1$  may be summarized as follows;

- 1) Strongly enhanced E2-transitions between the  $(j - 1)$ -states and the  $j$ -states. The enhancements of the E2-transitions are comparable (or somewhat stronger) to those of phonon transitions in neighbouring even-even nuclei.
- 2) The  $g$ -factors of the  $(j - 1)$ -states are nearly equal to (or slightly deviate from) those of the single-quasiparticle states with spin  $j$ .
- 3) Moderately hindered M1-transitions between the  $(j - 1)$ -states and the  $j$ -states. In some experiments, however, they are only weakly retarded.

Based on these characteristic experimental facts, we have proposed a new point of view on the structure of ACS,<sup>10)</sup> where the main component of the ACS is regarded as the dressed three-quasiparticle (3QP) modes which manifest themselves as relatively pure elementary excitation modes. The concept of dressed  $n$ -quasiparticle modes and the theory of quasiparticle-new-Tamm-Dancoff (NTD)-space spanned by these elementary excitation modes

has been developed by the present authors and by Kanasaki, Sakata and Takada.<sup>10,11)</sup> The quasiparticle-NTD space is constructed in a complete one-to-one correspondence with the quasiparticle-Tamm-Dancoff (TD)-space and, therefore, the introduced new collective modes (dressed 3QP modes) are reduced to the Tamm-Dancoff 3QP states in the limit of neglect of the ground-state correlations. Thus, the strongly enhanced E2-transitions which characterize the collective nature of ACS can be naturally explained, contrary to the Kisslinger's (Tamm-Dancoff) 3QP "intruder" states.<sup>12)</sup> In the proposed point of view, the significance of the ACS with spin  $j - 1$  in spherical odd-mass nuclei may be compared with that of phonon states with  $J^\pi = 2^+$  in even-even nuclei, in the sense that they are both *the typical phenomena of the collective excitation modes*. The well-known phonon modes are, in our classification, "dressed 2QP modes" which are described by the conventional 2QP-NTD method (RPA) and the "dressed 3QP modes" are nothing but the ACS under consideration.

Under the special physical condition of shell structure for the appearance of ACS, the creation operators of the physical dressed 3QP modes are constructed in terms of quasiparticle creation and annihilation operators as<sup>10)</sup>

$$Y_n^\dagger = \frac{1}{\sqrt{3!}} \sum_{\pi\rho\sigma} \{ \psi_n^p(\pi\rho\sigma) T_{3/2\ 3/2}(\pi\rho\sigma) + \varphi_n^p(\pi\rho\sigma) : T_{3/2\ 1/2}(\pi\rho\sigma) : \} \\ + \frac{1}{\sqrt{2}} \sum_{\pi\beta\gamma} \{ \psi_n^c(\pi\beta\gamma) a_\pi^\dagger a_\beta^\dagger a_\gamma^\dagger + \varphi_n^c(\pi\beta\gamma) a_\pi^\dagger \tilde{a}_\beta \tilde{a}_\gamma \}, \quad (1)$$

with the condition

$$\sum_\pi \psi_n^p(\pi\tilde{\pi}\sigma) = \sum_\pi \varphi_n^p(\pi\tilde{\pi}\sigma) = 0. \quad (2)$$

Here Greek letters ( $\pi, \rho, \sigma$ ) are used to specify the single-particle states in the large-spin, opposite parity level (such as  $g_{9/2}^+$  and  $h_{11/2}^-$ ) and Greek letters ( $\alpha, \beta, \gamma$ ) denote the other states (for both protons and neutrons). It should be noticed that the lowering of ( $j - 1$ )-states occurs in odd-mass nuclei in the region of spherical to transitional, when a level of large spin  $j$  with unique parity in the major shell is being filled. In eq. (1), the operators  $T_{3/2\ 3/2}(\pi\rho\sigma)$  are the quasi-spin tensors of rank 3/2 composed of quasi-particle trilinear products:

$$T_{3/2\ 3/2}(\pi\rho\sigma) = a_\pi^\dagger a_\rho^\dagger a_\sigma^\dagger, \quad (3) \\ T_{3/2\ 1/2}(\pi\rho\sigma) = \sqrt{\frac{1}{3}} \{ a_\pi^\dagger \tilde{a}_\rho \tilde{a}_\sigma + \tilde{a}_\pi a_\rho^\dagger \tilde{a}_\sigma + \tilde{a}_\pi \tilde{a}_\rho a_\sigma^\dagger \}.$$

The eigenmode operators  $Y_n^\dagger$  thus constructed *transfer* the seniority  $\Delta v = 3$  to the correlated ground state  $|0\rangle$  and create the dressed 3QP states  $|n\rangle$ . Under the basic approximation of the NTD method, it can be shown that the eigenmode operators  $Y_n^\dagger$  satisfy the *quasi-Fermion approximation*,

$$\langle 0 | \{ Y_{n'}, Y_n^\dagger \}_+ | 0 \rangle = \delta_{nn'}. \quad (4)$$

The equations of motion for the dressed 3QP modes were solved with the use of the pairing-plus-quadrupole (P + QQ) force in ref. 10). The excitation-energy systematics of the ACS, which have some similarity with those of  $2^+$  phonon states in the sequence of even-even isotopes, were reproduced very well in the numerical calculations.

§2. Now, let us discuss how to calculate the electromagnetic quantities in the framework of the proposed theory and whether the characteristics of the electromagnetic properties of the ACS mentioned before are explained consistently by the proposed point of view or not. Since the essence of our theory is to treat the (odd-mass nuclear) system within the “quasi-particle-NTD-subspace” (which is formed with orthogonal basis vectors consisting of the correlated ground state  $|0\rangle$ , the 1QP states  $|\alpha\rangle = a_\alpha^\dagger|0\rangle$  and the dressed 3QP states  $|n\rangle = Y_n^\dagger|0\rangle$ ), we must transcribe any physical operator  $\hat{O}_{\lambda\mu}$  (where  $\lambda$  means the rank of tensor) into the “quasiparticle-NTD-subspace.” The transcription can be done unambiguously,<sup>11)</sup> the result of which is

$$\hat{O}_{\lambda\mu} \Rightarrow \overset{\circ}{O}_{\lambda\mu} = \sum_{\alpha,\alpha'} \langle \alpha | \hat{O}_{\lambda\mu} | \alpha' \rangle a_\alpha^\dagger a_{\alpha'} + \sum_{n,n'} \langle n | \hat{O}_{\lambda\mu} | n' \rangle Y_n^\dagger Y_{n'} + \sum_{\alpha,n} \langle \alpha | \hat{O}_{\lambda\mu} | n \rangle \cdot (a_\alpha^\dagger Y_n + Y_n^\dagger a_\alpha). \tag{5}$$

The matrix elements of eq. (5) are evaluated by using the quasi-Fermion approximation (4);

$$\begin{aligned} \langle \alpha | \hat{O}_{\lambda\mu} | \alpha' \rangle &= \langle 0 | \{ a_\alpha, [\hat{O}_{\lambda\mu}, a_{\alpha'}^\dagger]_- \}_+ | 0 \rangle, \\ \langle \alpha | \hat{O}_{\lambda\mu} | n \rangle &= \langle n | \hat{O}_{\lambda\mu} | \alpha \rangle \\ &= \langle 0 | \{ a_\alpha, [\hat{O}_{\lambda\mu}, Y_n^\dagger]_- \}_+ | 0 \rangle \\ &= \langle 0 | \{ Y_n, [\hat{O}_{\lambda\mu}, a_\alpha^\dagger]_- \}_+ | 0 \rangle, \\ \langle n | \hat{O}_{\lambda\mu} | n' \rangle &= \langle 0 | \{ Y_n, [\hat{O}_{\lambda\mu}, Y_{n'}^\dagger]_- \}_+ | 0 \rangle. \end{aligned} \tag{6}$$

In the same way, our original Hamiltonian  $H$  is expressed, after the transcription, as

$$\overset{\circ}{H} = \sum_\alpha E_\alpha a_\alpha^\dagger a_\alpha + \sum_n \omega_n Y_n^\dagger Y_n + \sum_{n,\alpha} \bar{\chi}_n (Y_n^\dagger a_\alpha + a_\alpha^\dagger Y_n). \tag{7}$$

The third term of the effective Hamiltonian  $\overset{\circ}{H}$  represents the interaction between the different modes of elementary excitations, and comes from the  $H_Y$ -type (original) interactions which have not played any role in constructing the elementary excitation modes, contrary to the  $H_X$ - and  $H_V$ -type (original) interactions (Fig. 1). As long as the ACS with spin  $j - 1$  are regarded as relatively pure dressed 3QP modes, the third term can safely be dropped. In other words, the special condition to attenuate the effects of the third term is nothing but the condition to guarantee the appearance of the ACS in their most simple and pure form. We show at the first, therefore, the results on the E2-transitions and the magnetic moments of the ACS in the first-step approximation (by neglecting the third term), with the use of the  $P + QQ$  force. The effects of the third term (*i.e.*, the interplay of the dressed 3QP modes and the 1QP modes) will be discussed later in connection with the M1-transitions.

§3. The  $B(E2)$  from the ACS with spin  $I$  to the 1QP state with spin  $j$  is given by

$$\begin{aligned} B(E2; I \rightarrow j) &= \left| e_\tau Q(pp) \sqrt{C_I} \left\{ \psi_n(p^3) + \sqrt{\frac{1}{3}} \varphi_n(p^3) \right\} \right. \\ &\quad \left. + \sum_{b,c} e_\tau Q(bc) \{ \psi_n(p; bc) + \varphi_n(p; bc) \} \right|^2, \end{aligned} \tag{8}$$

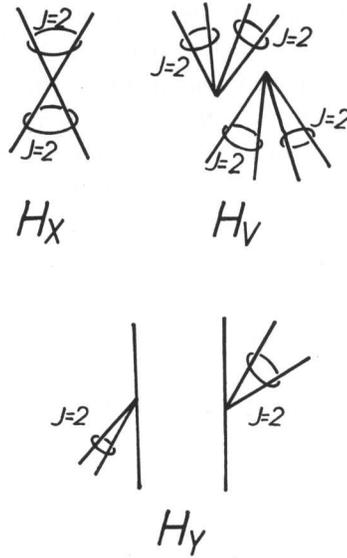


Fig. 1. Graphic representation of the matrix elements of the interaction.

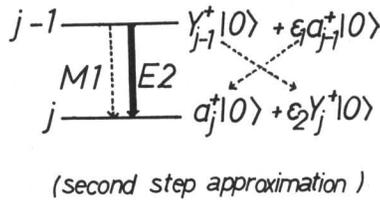
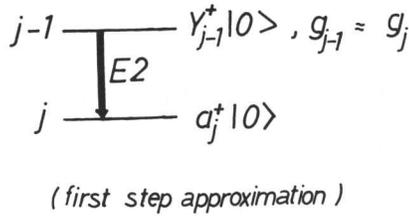


Fig. 2. Graphic representation of our basic states and their electromagnetic properties.

where

$$Q(bc) \equiv \frac{1}{\sqrt{10}} \langle b || r^2 Y_2 || c \rangle \cdot (u_b v_c + v_b u_c),$$

$$C_I \equiv 1 + 10 \left\{ \begin{matrix} jj2 \\ jI2 \end{matrix} \right\} - \delta_{jI} \frac{20}{(2j)^2 - 1}.$$

In eq. (8), the 3QP-correlation amplitudes,  $\psi_n(p^3)$ ,  $\psi_n(p; bc) \dots$  etc, are related with those defined in eq. (1), through

$$\begin{aligned}\psi_n(\pi\rho\sigma) &= \sum_{J=\text{even}} \psi_n(j^2(J)j\}j^3I)\langle JjMm_\sigma|IK\rangle\langle jjm_\pi m_\rho|JM\rangle, \\ \psi_n(\pi\beta\gamma) &= \psi_n(p; bc)\langle 2jMm_n|IK\rangle\langle j_b j_c m_\beta m_\gamma|2M\rangle, \\ \psi_n(j^2(2)j\}j^3I) &= \sqrt{C_I/3} \cdot \psi_n(p^3), \dots \text{etc.}\end{aligned}\quad (9)$$

It is interesting to note that, in the P + QQ force, the final results can be expressed in terms of only the special amplitudes with intermediate angular momentum  $J = 2$ . It is also noted that formally eq. (8) has the similar structure as the corresponding equation obtained by the conventional RPA in even-even nuclei, in spite of the essential difference due to the incorporation of the 3QP correlations. For the E2-transition from the excited ACS to the 1QP states, we therefore expect the well-known enhancement associated with the structure of eq. (8). In particular, we have the usual relation: the lower the excitation energy of the ACS, the larger the  $B(E2)$  values. *Such an enhancement, caused by the collective ground-state correlations due to the QQ-force is a direct and natural consequence of the present theory.*

With the use of eqs. (5) and (6), the magnetic moments of the ACS with spin  $I$  is given by

$$\mu = g_I \cdot I, \quad (10)$$

with

$$\begin{aligned}g_I &= g_p^{(0)} + \frac{I(I+1) + j(j+1) - 6}{2I(I+1)} \cdot g_p^{(1)} \\ &\quad + \frac{I(I+1) + 6 - j(j+1)}{2I(I+1)} \cdot g_c.\end{aligned}\quad (11)$$

The partial  $g$ -factors in this equation are

$$g_p^{(0)} = g_p \cdot \{\psi_n^2(p^3) - \varphi_n^2(p^3)\}, \quad (12)$$

$$g_p^{(1)} = g_p \cdot \sum_{bc} \{\psi_n(p; bc)^2 - \varphi_n(p; bc)^2\} \quad (13)$$

and

$$\begin{aligned}g_c &= \sqrt{\frac{10}{3}} \sum_{abc} \langle b\|\mu\|c\rangle \cdot (u_b u_c + v_b v_c) \cdot \begin{Bmatrix} 2 & j_c & j_a \\ j_b & 2 & 1 \end{Bmatrix} \\ &\quad \times [\psi_n(p; ca)\psi_n(p; ab) - \varphi_n(p; ca)\varphi_n(p; ab)],\end{aligned}\quad (14)$$

respectively. Here  $g_p$  means the  $g$ -factor of a single-particle in the high spin, unique parity level  $j$ . The meaning of each term in eq. (11) is clear. The first term,  $g_p^{(0)}$ , comes from the quasiparticles in the unique parity level  $j$ . If we restrict ourselves only within the unique parity level  $j$ , which is being filled,  $g_p^{(0)}$  becomes equal to  $g_p$  (because in this case  $\psi_n^2(p^3) - \varphi_n^2(p^3) = 1$ ). The second and third terms are of the same form as in the Lande-formula; the second term comes from the odd-quasiparticle in the level  $j$  and the third term comes from the quasi-particles in the core, respectively. It is important to notice that the contributions from the quasi-particles in the core, (the amplitude of which is represented by  $\psi_n(p; ab)$  and  $\varphi_n(p; ab)$ ), are accompanied with the kinematical factor, which becomes small especially for  $I = j - 1$ . This means that, in some situations, the quasiparticles in the core give rise to only small effects to the total  $g$ -factor and the magnetic moments of the ACS are determined

mainly by the quasi-particles in the unique parity level  $j$ . Therefore, the observed value of  $g_I$  nearly equal to  $g_p$  does not necessarily mean the simple  $(j^n)_I$ -configuration. *Even if the wave functions under consideration be far from those of simple  $(j^n)_I$ -configurations, the magnetic moments give us nearly the same values in experiments.*

In the first-order approximation with the P + QQ force, in which the ACS with  $I = j - 1$  are regarded as relatively pure dressed 3QP modes, the M1-transition between the ACS and the 1QP states with spin  $j$  is forbidden, as is easily seen by our construction of the eigenmode operator (1) with correlation amplitudes (9):

$$B(\text{M1}; I \rightarrow j) = 0. \quad (15)$$

The attenuation of the M1-transitions is indeed observed in experiments<sup>2)</sup> and is a sensitive criterion for the purity of the ACS as the dressed 3QP states. In some experiments, however, it is only weakly retarded.<sup>4)</sup> In order to explain the small M1-transitions, therefore, we must consider the coupling effects of the dressed 3QP modes with 1QP modes.<sup>13)</sup> The interplay of the dressed 3QP modes with the 1QP modes is originated from the third term of the effective Hamiltonian (7) in the quasiparticle-NTD-space under consideration. In the P + QQ force model, the coupling strength  $\bar{\chi}_n$  is given as follows;

$$\begin{aligned} \bar{\chi}_n = & -\sqrt{\frac{1}{2I+1}} \cdot \chi \langle p' \| r^2 Y_2 \| p \rangle (u_{p'} u_p - v_{p'} v_p) \delta_{j_p, I} \\ & \times \left[ Q(pp) \sqrt{C_I} \left\{ \psi_n(p^3) + \sqrt{\frac{1}{3}} \varphi_n(p^3) \right\} \right. \\ & \left. + \sum_{b,c} Q(bc) \{ \psi_n(p; bc) + \varphi_n(p; bc) \} \right], \end{aligned} \quad (16)$$

where  $\chi$  is the (original) strength of the quadrupole-force. The characteristic of the coupling term is its inclusion of the  $(u_{p'} u_p - v_{p'} v_p)$ -factor, which comes from the (original)  $H_Y$ -type interaction. In the special physical situations in which high-spin, unique parity level  $j_p$  is half-filled, we have

$$u_p^2 - v_p^2 \approx 0 \quad (\text{for } p' = p).$$

Furthermore, since a single-particle level  $p'$  (which has the same parity with the level  $p$ ) with spin  $j_{p'} = I \neq j_p$  does not exist in the same major shell and is lying in the next upper major shell, the coupling effect is expected to be rather small.

Including the coupling effects, we obtain an expression for the M1-transition under consideration:

$$B(\text{M1}; I \rightarrow j) = \frac{3}{4\pi} \cdot \frac{1}{2I+1} \cdot |\varepsilon_1 \langle p \| \mu \| p' \rangle_{\text{q.p.}} + \varepsilon_2 \langle n \| \mu \| n' \rangle_{\text{coll.}}|^2, \quad (17)$$

where

$$\langle p \| \mu \| p' \rangle_{\text{q.p.}} \equiv \langle p \| \mu \| p' \rangle \cdot (u_p u_{p'} + v_p v_{p'})$$

and  $\langle n \| \mu \| n' \rangle_{\text{coll.}}$  is the reduced matrix element of  $\langle 0 | Y_n \mu Y_n^\dagger | 0 \rangle$ . The first term of eq. (17) represents the contribution due to the admixture of the 1QP modes with spin  $j_{p'} = j_p - 1$  (from the upper major shell) to the ACS with spin  $I = (j_p - 1)$  and usually small. The second

Table I.  $B(E2)$  values for the transition from the ACS to the 1QP states. The calculated values are listed in unit of  $e^2 \times 10^{-50} \text{ cm}^4$  for polarization charge  $\alpha = 0.5$ .

a) ref. 2, b) ref. 3, c) ref. 4, d) ref. 9.

$B(E2; 9/2^- \rightarrow 11/2^-)$			$B(E2; 7/2^+ \rightarrow 9/2^+)$		
Nucleus	Cal.	Exp.	Nucleus	Cal.	Exp.
$^{113}\text{Cd}$	9.8		$^{97}\text{Tc}$	8.3	
$^{125}\text{Te}$	10.5	{ 9.3 <sup>a)</sup> 11.5 $\pm$ 0.5 <sup>b)</sup>	$^{99}\text{Tc}$	12.0	23 $\pm$ 12 <sup>c)</sup> 13.5 $\pm$ 1.5 <sup>d)</sup>
$^{127}\text{Te}$	8.2		$^{99}\text{Rh}$	9.4	
$^{129}\text{Te}$	6.2		$^{101}\text{Rh}$	14.9	
$^{131}\text{Xe}$	15.3		$^{85}\text{Sr}$	6.1	
			$^{83}\text{Kr}$	13.0	

 Table II. Gyromagnetic ratio  $g_I$  for the ACS with  $I = j - 1$ . The calculated values are listed in unit of n.m. for effective spin  $g$ -factor  $g_s^{\text{eff}} = 0.6 g_s$ . For the values of  $g_p$ , the following experimental values are directly adopted;

 $h_{1/2}^-$ -odd-neutron:  $g_p = -0.19$  ( $^{113}\text{Cd}$ )

 $g_{3/2}^-$ -odd-neutron:  $g_p = -0.22$  ( $^{83}\text{Kr}$ )

 $g_{5/2}^-$ -odd-neutron:  $g_p = 1.37$  ( $^{93}\text{Nb}$ )

a) ref. 5, b) ref. 6, c) ref. 7, d) ref. 8.

$g_{9/2^-}$			$g_{7/2^+}$		
Nucleus	Cal.	Exp.	Nucleus	Cal.	Exp.
$^{113}\text{Cd}$	-0.26		$^{97}\text{Tc}$	1.41	
$^{125}\text{Te}$	-0.21	{ -0.204 $\pm$ 0.007 <sup>a)</sup> -0.202 $\pm$ 0.016 <sup>b)</sup> -0.15 $\pm$ 0.02 <sup>c)</sup>	$^{99}\text{Tc}$	1.40	
			$^{99}\text{Rh}$	1.39	
$^{127}\text{Te}$	-0.22		$^{101}\text{Rh}$	1.37	
$^{129}\text{Te}$	-0.23		$^{85}\text{Sr}$	-0.24	
$^{131}\text{Xe}$	-0.22		$^{83}\text{Kr}$	-0.22	-0.268 $\pm$ 0.001 <sup>d)</sup>

 Table III.  $B(M1)$  values for the transitions from the ACS to the 1QP states. The calculated values are listed in unit of (n.m.)<sup>2</sup> for  $g_s^{\text{eff}} = 0.6 g_s$ .

a) ref. 2, b) ref. 3, c) ref. 4.

$B(M1; 9/2^- \rightarrow 11/2^-)$			$B(M1; 7/2^+ \rightarrow 9/2^+)$		
Nucleus	Cal.	Exp.	Nucleus	Cal.	Exp.
$^{113}\text{Cd}$	0.16		$^{97}\text{Tc}$	0.038	
$^{125}\text{Te}$	0.0001	{ 0.0053 <sup>a)</sup> (0.0065 <sup>b)</sup> $\pm$ 0.0003	$^{99}\text{Tc}$	0.033	0.076 $\pm$ 0.009 <sup>c)</sup>
			$^{99}\text{Rh}$	0.0003	
$^{127}\text{Te}$	0.0026		$^{101}\text{Rh}$	0.0011	
$^{129}\text{Te}$	0.014		$^{85}\text{Sr}$	0.0025	
$^{131}\text{Xe}$	0.026		$^{83}\text{Kr}$	0.0080	

term comes from the admixture of the dressed 3QP mode with spin  $j_p$  to the 1QP state with spin  $j_p$ . Because the second term contains the  $(u_p^2 - v_p^2)$ -factor through the mixing amplitude  $\varepsilon_2$ , the value depends quite sensitively on the single-particle energy adopted and can become large as one moves away from the special physical situation (for the appearance of ACS) mentioned before.

In Tables I ~ III, the calculated values on  $B(E2)$ ,  $g$ -factors and  $B(M1)$  are compared with experimental values in some examples. In this calculation, the same values of the pairing-force strength and of the single-particle energies with Uher and Sorensen<sup>14)</sup> were used and the quadrupole-force strength  $\chi$  was determined to reproduce the excitation energies of the ACS with spin  $j - 1$ . It is seen that the essential character of the various electromagnetic properties of the ACS has been explained in a unified way by the present theory, if not in fine detail.

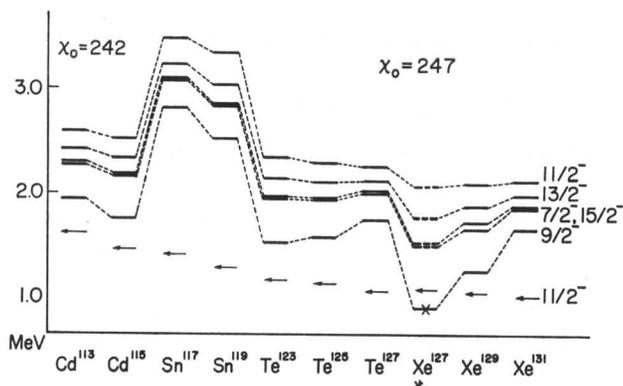
### References

- 1) L. S. Kisslinger and R. A. Sorensen: Rev. mod. Phys. **35** (1963) 853.
- 2) T. Inamura: J. Phys. Soc. Japan **24** (1968) 1.
- 3) A. Märelius *et al.*: Nuclear Phys. **A148** (1970) 433.
- 4) J. McDonald, A. Bäcklin and S. G. Malmkog: Nuclear Phys. **A162** (1971) 365.
- 5) D. W. Cruse, K. Johansson and E. Karlson: Nuclear Phys. **A154** (1970) 369.
- 6) E. Knapek *et al.*: Phys. Letters **29B** (1969) 581.
- 7) M. Rots, R. Silverans and R. Coussemant: Nuclear Phys. **A170** (1971) 240.
- 8) L. E. Campbell, G. J. Perlow and M. A. Grace: Phys. Rev. **178** (1969) 1728.
- 9) P. D. Bond, E. C. May and S. Jha: Nuclear Phys. **A179** (1972) 389.
- 10) A. Kuriyama, T. Marumori and K. Matsuyanagi: Progr. theor. Phys. **45** (1971) 784 and *ibid.* **47** (1972) 498.
- 11) N. Kanesaki, F. Sakata, T. Marumori and K. Takada: preprints and to be published.
- 12) L. S. Kisslinger: Nuclear Phys. **78** (1966) 341.
- 13) H. Ikegami and M. Sano: Phys. Letters **21** (1966) 323 and presented at this conference V-23.
- 14) R. A. Uher and R. A. Sorensen: Nuclear Phys. **86** (1966) 1.

### Discussion

H. YAMAMURA (Kyoto Univ.): Usually the low lying states of spherical odd nuclei are investigated in terms of the phonon-quasiparticle-coupling model given by Kisslinger and Sorensen. But your description seems to be different from theirs. What is the essential difference?

MATSUYANAGI: This figure clearly shows the essential difference between our theory and the Kisslinger-Sorensen theory. The lowering of energies of the  $I = j - 1$  states cannot be explained by the Kisslinger-Sorensen theory, *i.e.* the conventional phonon-quasiparticle-coupling theory completely breaks down in reproducing such low-lying anomalous coupling states. However, we have obtained the result, shown in the figure, by using just the same parameters as Kisslinger and Sorensen have used. The reason is that, in the conventional



Calculated excitation energies of dressed three quasi-particle modes. Single quasi-particle energies in orbit  $\pi$  are written by arrows. It should be noticed that all energies are measured from the ground states of their modes. Thus the differences of these energies are those which correspond to the spectra of odd-mass nuclei. The symbol "x" means that the calculated energy of  $(j - 1)$  state becomes smaller than the single quasi-particle energy  $E_{\pi}$ . In this case other angular momentum states are written by broken lines. The nuclei whose  $(j - 1)$  states are found below  $j$  states are denoted by the asterisk\*.

phonon-quasiparticle-coupling theory, elementary excitation modes of spherical odd-mass nuclei are assumed to be one quasiparticle modes, one phonon modes, and two phonon modes, etc. On the contrary, in our theory, the elementary excitation modes are considered as one quasiparticle modes, the 'dressed' three quasiparticle modes and the 'dressed' five quasiparticle modes, etc. In the conventional phonon-quasiparticle-coupling theory, the three quasiparticle correlations based on the Pauli Principle between the odd quasiparticle and the quasiparticles composing the phonon are completely neglected. Such effects are fully taken into account in our theory.

A. ARIMA (Stony Brook and Tokyo): I think that the main difference between your theory and the Kisslinger-Sorensen theory is in the Pauli principle. In your method, I think, the Pauli principle is correctly taken into account, but in the Kisslinger-Sorensen theory the effect of the Pauli principle is ignored. That is a most important difference. Is this correct?

MATSUYANAGI: Yes.