JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN VOL. 34, SUPPLEMENT, 1973
PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON NUCLEAR MOMENTS AND NUCLEAR STRUCTURE, 1972

VI.b. Renormalization of the E0, M1 and E2 Operators

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a) The E0 Transitions and Isotope Shifts

Due to fortuitous circumstances the transition rate of the 0_2 state in ^{206}Pb at 1.07 MeV is dominated by the E0 process to the ground state. This can be understood in terms of True's wave functions which allow two neutron holes in the $p_{1/2}$, $f_{5/2}$, $p_{3/2}$, $f_{7/2}$, $i_{13/2}$ and $h_{9/2}$ orbits. One finds that with these wave functions the $B(\mathrm{E2}, 0_2 \to 2_1)$ is strongly supressed. A slight modification of his wave function will cause this transition branch to vanish.

The lifetime of the 0_2 state, as measured at University of Washington by Adelberger, Tape and Burch is approximately 10^{-9} seconds.

How can we get a finite E0 rate? After all, the operator is $\sum_{\text{protons}} r^2(i)$ and hence should vanish for neutron hole states. Why not assign an effective charge for the E0 process just as one often does for the E2 rates?

If we do this we can use True's wave functions. But, for the sake of simplicity, let us for the moment consider two component wave functions. Everything we say will carry through to the more general case. Thus

$$|0_1\rangle = a|p_{1/2}^{-2}\rangle_0 + b|f_{5/2}^{-2}\rangle_0 \ ,$$

and

$$|0_2\rangle = -b|p_{1/2}^{-2}\rangle_0 + a|f_{5/2}^{-2}\rangle_0 \ .$$

The E0 matrix element is then equal to

$$-2ab[e(p_{1/2})\langle p_{1/2}|r^2|p_{1/2}\rangle - e(f_{5/2})\langle f_{5/2}|r^2|f_{5/2}\rangle] \ .$$

We have purposely distinguished the effective charges of the $p_{1/2}$ and $f_{5/2}$ orbits. The reason for this is that with harmonic oscillator wave functions, the mean square radius is the same for all orbits in a given major shell. Hence $\langle p_{1/2}|r^2|p_{1/2}\rangle = \langle f_{5/2}|r^2|f_{5/2}\rangle$. With Wood-Saxon wave functions this is no longer true, but we have checked that in this case. They are sufficiently close to each other, so one cannot obtain any significant value for the E0 matrix

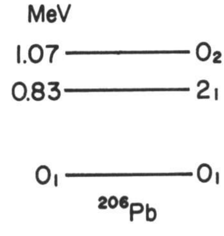

Fig. 1

element. The simplest explanation of the strong E0 rate is that the E0 effective charge is state dependant $e(\mathrm{p}_{1/2}) \neq e(\mathrm{f}_{5/2})$.

The effective charge can be obtained empirically from the isotope shifts. These are usually expressed in terms of standard shifts, which correspond to the law $R = 1.2A^{1/3}$. For a standard shift the change in mean square radius as one adds a nucleon to a closed shell nucleus

$$\delta r^2 = 3/5r_0^2[(A+1)^{2/3} - A^{2/3}] \approx 2r_0^2/5A^{1/3} \ .$$

Experimenta ly the ^{209}Pb – ^{208}Pb difference is 1 standard shift; the ^{208}Pb – ^{207}Pb difference is about 1/2 standard shift. Thus the *effective charge* of a $\mathrm{p}_{1/2}$ hole is

$$e(\mathrm{p}_{1/2}) = 1/2\ (2r_0^2/5A_{1/3})/\langle \mathrm{p}_{1/2}|r^2|\mathrm{p}_{1/2}\rangle \ .$$

The value is 0.107 (the E2 effective charge is about 0.9). In ^{209}Pb, the $\mathrm{g}_{9/2}$ effective charge is 0.18.

In order to get the strong E0 rate in ^{206}Pb we require that the effective charge for the $\mathrm{f}_{5/2}$ orbit is

$$\begin{aligned} e(\mathrm{f}_{5/2}) &= 0.1 + 0.04 = 0.14 \\ \text{or} \qquad &= 0.1 - 0.04 = 0.06 \ . \end{aligned}$$

We thus predict an *isomeric* shift in ^{207}Pb of the first excited state $(\mathrm{f}_{5/2}^{-1})$ relative to the $\mathrm{p}_{1/2}^{-1}$ ground state.

Our state-dependent hypothesis requires experimental confirmation or denial. I hope to make some quantitative theoretical calculations.

b) The M1 Operator

The magnetic moment operator is the sum of a one body part and a two body part coming from meson exchange:

$$\boldsymbol{\mu} = g_l\boldsymbol{l} + g_s\boldsymbol{s} + \boldsymbol{M}(12) \ .$$

It is ironical that in the nucleus the two body operator behaves like a one body operator, and the one body operator behaves like a one and two body operator. Let me explain.

Yamazaki and Nomura showed very clearly that, empirically, the meson exchange effects can be taken into account by replacing g_l by $g_l \pm 0.1$ for proton and neutron. But this is a one body renormalization. There is at present no experimental information in large nuclei which demands an explicit two body operator.

If we stay in a restricted shell model space, *i.e.* a single j shell, then to obtain good results one must not only renormalize the one body part of the operator $g_l\boldsymbol{l} + g_s\boldsymbol{s}$, but one must introduce an explicit two body operator. This two body part can be calculated in terms of configuration mixing as was done by Horie and Arima.

As an example, consider the $N = 28$ nuclei. I refer now to work of Goode and myself published in March 1972 issue of Particles and Nuclei. The nuclei I wish to mention have 1, 2, 3, 5 and 7 proton holes relative to ^{56}Ni. They are ^{55}Co, ^{54}Fe, ^{53}Mn, ^{51}V and ^{49}Sc.

With particles of one kind in a given j shell we have the following theorem:

$\boldsymbol{\mu}$ can be replaced by $g\boldsymbol{J}$.

Hence

a) All g factors in the shell are the same.

b) All M1 rates vanish.

To get varying g factors and finite rates we introduce the two body magnetic moment operator $\boldsymbol{\delta\mu}(12)$. It is convenient to note that this operator can be written as the product of a two body scalar and the angular momentum operator: $\boldsymbol{\delta\mu}(12) = U(12)[\boldsymbol{J}(1) + \boldsymbol{J}(2)]$. This is the most *general* form in a single j shell of protons. Even the meson operators must have this simple form.

To learn about two body operator we consider the two hole system ^{54}Fe. In the $f_{7/2}^{-2}$ configuration we have $J = 0$, 2, 4 and 6. The deviation of the g factors of ^{54}Fe from ^{55}Co ground state are given by

$$\delta\mu^I = (j^{-2})^I U(12)(j^{-2})^I \, I \, (I = 2, 4, 6) \ .$$

The relevant diagram for calculating $\delta\mu^I$, due to configuration mixing, à la Arima and Horie, is shown. Note that the intermediate particle *must* be an $f_{5/2}$ proton. This is reasonable because the two body operator reflects the fact that as one deletes protons from ^{56}Ni, less protons can be excited into the $f_{5/2}$ orbit and so there should be less quenching.

But this tells us that the one pion exchange process is unimportant for this diagrams, since two protons can only interchange a neutral pion. We must qualify the above remark slightly. The diagram is proportional to $(g_l - g_s)$. We *should* change g_l from 1 to 1.1 à la Yamazaki and Nomura. But, as was pointed out by Arima, g_s is much larger than g_l, so the effect of this is very small.

A two body operator may appear complicated but it leads to simple predictions.

1) The ground state magnetic moments of the odd nuclei (^{55}Co, ^{53}Mn, ^{51}V, ^{49}Sc) should lie on a straight line.

2) The M1 rates for 3 holes (^{53}Mn) should be equal to the corresponding rates for 3 particles (^{51}V).

Unfortunately, the ^{49}Sc moment is not known, and ^{55}Co is poorly measured. In fact, I believe that for the ^{55}Co case it is better, until and experiment is done, to use the moment of ^{57}Co and ^{59}Co which are nearly the same. Mavromatis, Brown and myself calculated that all 3 nuclei should have essentially the same moment.

Horie has discussed the ground state moments in a preceeding talk so I will be brief. The moments ^{51}V, ^{53}Mn and ^{59}Co do not lie on a straight line. Hence effects beyond the two body are being seen. Goode and I did a matrix diagonalization in the space $f_{7/2}^{-n}$ and $f_{7/2}^{-(n+1)}$ j' with $j' = p_{3/2}, f_{5/2}$ or $p_{1/2}$. This caused the ^{55}Co moment to get closer to Schmidt and gave

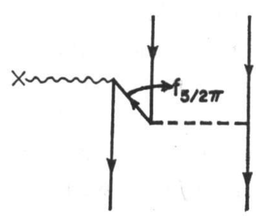

Fig. 2

a small deviation from linearity. However, if we draw a line through the two well determined moments ^{51}V and ^{53}Mn, then this line has a much smaller slope than the average slope obtained in this calculations. From other contributions to this conference, this seems to be a universal problem—the calculated slope is too steep.

Horie obtains a deviation from linearity by simply raising the energy of the $T = 1$ part of the M1 Giant resonance $(f_{5/2}\, f_{7/2}^{-1})^{J=1}$ relative to the $T = 0$ part. This is much simpler than our procedure. On the other hand we include the effect of $p_{3/2}$ mixing. A detailed comparison of the two approaches would be of interest.

A yet unsolved problem is the fact that the corresponding slope for the calcium isotopes, or determined by ^{41}Ca and ^{43}Ca, is steeper than the slope for the $N = 28$ nuclei. This is at first surprising because $g_l - g_s$ is larger for protons than for neutrons. Perhaps there is more deformed admixture in ^{43}Ca than in ^{41}Ca.

We have also extended the calculation to M1 rates. Before discussing this, let us anticipate that M1 rates will be very small due to $f_{5/2}$ admixture or two body moments. Suppose that the two body moment $U(12)\boldsymbol{J}$ is such that $\langle (j^{-2})^I U(12)(j^{-2})^I \rangle$ is independent of I ($I = 2, 4, 6$). Set it equal to a constant U. Then $\sum_{i<j} \delta\boldsymbol{\mu}(ij) = U(n-1)\boldsymbol{J}$. In other words the two body operator has collapsed into the angular momentum operator times the number of particles minus one.

In that case g factors will still vary but M1 rates will vanish. We note that with realistic forces we do indeed obtain the result that $U(12)$ is nearly constant.

I noticed from preceeding reports on protons in the $h_{9/2}$ shell that U is nearly constant in ^{210}Po. We therefore expect that the M1 rates in ^{211}At will be very small.

Since two body M1 rates are supressed we might expect a violation of the equality of M1 rates for particle and holes. This has indeed been observed both by Bizetti *et al.* from Italy and Goodman and Donahue from Arizona. Although for the $5/2_1 \to 7/2_1$ transition the M1 rates are nearly equal in both nuclei, there is a factor of 10 or more difference for the $3/2_1 \to 5/2_1$ transition. In the matrix diagonalization calculation of Goode and myself we are able to explain this difference.

c) E2 Moments

Here I will be very brief. I report on a calculation of Richard Sharp and myself.

In calculating the effective charges of ^{17}O, using standard forces, even realistic ones we obtained a state dependence. The effective charge for the quadrupole moment $d_{5/2} \to d_{5/2}$ is greater by about $\sqrt{2}$ than the effective charge for the B(E2) $(s_{1/2} \to d_{5/2})$. But in Bohr and Mottelson Vol. 1 the effective charges are analyzed to be nearly the same.

However, we find with newer density dependent interactions, such as Vautheir-Brink, or Moszkowski-Ehlers, the two effective charges are nearly the same. We realize, though, that great experimental precision is required to distinguish between state dependence and state independence.

Discussion

S. FALLIEROS (Brown Univ.): In your abstract, you mention something about negative monopole effective charges. Can you help us understand this peculiarity?

ZAMICK: The point is that, with an ordinary force like a δ-force or a Kuo-Brown force etc., if we add a neutron to ^{208}Pb, we would predict that ^{209}Pb would have a smaller charge radius than ^{208}Pb. That means a negative effective charge. But that is the wrong answer; it doesn't agree with experiment. We need a density dependent force (as shown first by Barrett, I think) in order to get positive isotope shifts.

B. SORENSEN (Copenhagen): I don't understand why your $e_{\text{eff}}(\text{E0})$ and $e_{\text{eff}}(\text{E2})$ in ^{206}Pb differ so much. Both involve matrix elements of r^2. In a calculation of E0 and E2 transitions in rate earth nuclei, Ascuitto and I found that the same $e_{\text{eff}}(\text{E0}) = e_{\text{eff}}(\text{E2}) \simeq 0.3$ works for all observed states.

ZAMICK: Perhaps it is because I am close to a closed shell. It could be that the effective E0 charge increases as you go to the deformed nuclei. Perhaps we are misunderstanding each other's definition.

I. HAMAMOTO (München): Could you say something about what kind of effect your renormalization has on the form factor of states (for example the form factor which one can obtain from electron scattering), especially when you introduce the density-dependent interaction?

ZAMICK: No, we haven't looked into this question.