JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN VOL. 34, SUPPLEMENT, 1973
PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON NUCLEAR MOMENTS AND NUCLEAR STRUCTURE, 1972

VI.g. Meson-Exchange Currents and ^{3}H, ^{3}He Magnetic Moments*

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(Presented by Y. E. Kim)

It is shown that the discrepancy between the experimental ^{3}H, ^{3}He isovector magnetic moment and the single-particle value is very nearly accounted for by two-nucleon one-pion-exchange contributions. About two thirds of the discrepancy is given by trinucleon S–D matrix elements.

In this paper, we show that two-body one-pion-exchange contributions to the isovector trinucleon magnetic moment μ_V account for most of the discrepancy between the experimental value and the contribution coming from the single-particle operator

$$M^{(1)} = \sum_{i=1}^{3}\left[\frac{1}{2}(1-\tau_z(i))\mu_n\boldsymbol{\sigma}(i) + \frac{1}{2}(1+\tau_z(i))(\mu_p\boldsymbol{\sigma}(i) + \boldsymbol{L}(i))\right],$$

$$\mu_p = 2.793 \text{ n.m.},$$

$$\mu_n = -1.913 \text{ n.m.} \tag{1}$$

In the case of the isoscalar magnetic moment μ_S, it is shown that the one-pion-exchange contribution associated with the $\rho\pi\gamma$ vertex accounts for about one half the small discrepancy.

We use trinucleon wave functions, which are derived from an exact solution[1] of the Faddeev equations[2–5] for a nucleon-nucleon interaction (effective in the $^1S_0$ and $^3S_1 - {}^3S_1$ states) given by the Reid soft-core potential.[6] The trinucleon binding energy, in the absence of Coulomb effects, is 6.7 MeV and the percentages of $^2S_{\frac{1}{2}}$ (totally symmetric spatial part), $^2S'_{\frac{1}{2}}$ (mixed symmetry spatial part) and $^4D_{\frac{1}{2}}$ are 89.7%, 1.7% and 8.6% respectively. This solution has recently been confirmed by another calculation based on the coordinate space Faddeev equations.[7]

Graphs for the two-body contributions included in our calculations of μ_V and μ_S are given in Fig. 1. Explicit expressions for these contributions were derived in ref. 8. The pionic current (Fig. 1(a)) and pair excitation current (Fig. 1(b), (c)) terms are equivalent to those derived by Villars.[5] The vertex correction terms associated with s-channel singularities of the $\gamma N \rightarrow \pi N$ amplitude (Fig. 1(d), (e)) were calculated using the theory of Chew, Goldberger, Low and Nambu.[10] The contributions associated with t-channel singularities of the $\gamma N \rightarrow \pi N$ amplitude (Fig. 1(f)) were calculated using coupling constants $g_{\rho\pi\gamma}$ and $g_{\omega\pi\gamma}$, obtained from the ρ and ω radiative decay widths, and $g_{\rho NN}$ and $g_{\omega NN}$, obtained from the ρ and ω leptonic decay widths and the vector-meson-dominance model for nucleon electromagnetic form factors.

* Research supported in part by the United States National Science Foundation (Y.E.K. and E.P.H.) and Atomic Energy Commission (A.T.).

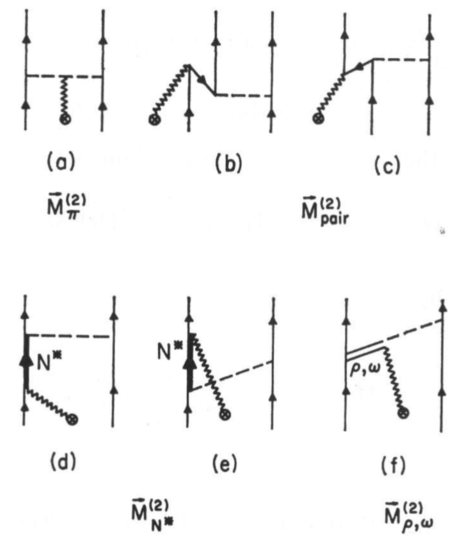

Fig. 1. Graphs for the two-body magnetic moment contributions included in our calculations of μ_V and μ_S.

We have not included "recoil" and "wave function normalization" contributions[8] to $\mu_{V,S}$ in our calculations. They are very model dependent and are hopefully small compared to the contributions indicated in Fig. 1. They were calculated in ref. 8 using an unperturbed Fock basis for the representation of two-nucleon states. Their contribution to μ_V was about 0.05 n.m. However, there is some indication that these contributions should cancel almost exactly if one uses the Heitler-London approach of Cutkosky.[11,12]

The matrix elements of the two-nucleon operators, in the relative-momentum basis with normalization $\langle \boldsymbol{p}'|\boldsymbol{p}\rangle = (2\pi)^3\delta(\boldsymbol{p}' - \boldsymbol{p})$, are

$$\langle \boldsymbol{p}'|\boldsymbol{M}^{(2)}_{\pi} + \boldsymbol{M}^{(2)}_{\text{pair}}|\boldsymbol{p}\rangle = \frac{\mathrm{M}}{\mathrm{m}_\pi^2}4\pi f^2_{\pi\mathrm{NN}}(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_z(q^2 + \mathrm{m}_\pi^2)^{-2}$$
$$\times [(\mathrm{m}_\pi^2 - q^2)\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j + 2(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j)\cdot \boldsymbol{q}\boldsymbol{q}]$$
$$\boldsymbol{q} = \boldsymbol{p}' - \boldsymbol{p},\ f_{\pi\mathrm{NN}} = \frac{g_r}{\sqrt{4\pi}}\left(\frac{\mathrm{m}_\pi}{2\mathrm{M}}\right),$$
$$g_r = 13.6; \tag{2}$$

$$\langle \boldsymbol{p}'|\boldsymbol{M}^{(2)}_{\mathrm{N}^*}|\boldsymbol{p}\rangle = \frac{2\pi}{3}(\mu_\mathrm{p} - \mu_\mathrm{n})(q^2 + m_\pi^2)^{-1}[h_1(0)(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_z((\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j)\cdot \boldsymbol{q}\boldsymbol{q} - q^2(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j))$$
$$+ 2h_2(0)(\boldsymbol{\tau}_i + \boldsymbol{\tau}_j)_z(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\cdot \boldsymbol{q}\boldsymbol{q} + 2h_2(0)(\boldsymbol{\tau}_i - \boldsymbol{\tau}_j)_z(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\cdot \boldsymbol{q}\boldsymbol{q}],$$
$$h_1(0) = 0.074\ \mathrm{m}_\pi^{-3},\quad h_2(0) = 0.066\ \mathrm{m}_\pi^{-3}; \tag{3}$$

$$\langle \boldsymbol{p}'|\boldsymbol{M}^{(2)}_{\omega}|\boldsymbol{p}\rangle = \frac{g_r g_{\omega\pi\gamma} g_{\omega\mathrm{NN}}}{2\mathrm{m}_\omega}[(q^2 + \mathrm{m}_\omega^2)\,(q^2 + \mathrm{m}_\pi^2)]^{-1}$$
$$\times [(\boldsymbol{\tau}_i + \boldsymbol{\tau}_j)_z(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\cdot \boldsymbol{q}\boldsymbol{q} + (\boldsymbol{\tau}_i - \boldsymbol{\tau}_j)_z(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\cdot \boldsymbol{q}\boldsymbol{q}],$$
$$g_{\omega\pi\gamma} = 0.758,\quad g_{\omega\mathrm{NN}} = 6.82; \tag{4}$$

$$\langle \boldsymbol{p}'|\boldsymbol{M}_\rho^{(2)}|\boldsymbol{p}\rangle = \frac{g_r g_{\rho\pi\gamma} g_{\rho\text{NN}}}{\text{m}_\rho}[(q^2+\text{m}_\rho^2)(q^2+\text{m}_\pi^2)]^{-1} \times (\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot\boldsymbol{q}\boldsymbol{q}\ ,$$

$$g_{\rho\pi\gamma} = 0.448,\ g_{\rho\text{NN}} = 2.56. \tag{5}$$

μ_S and μ_V are given by the $\mathscr{J} = \mathscr{J}_z = \frac{1}{2}$ matrix elements

$$\mu_{\text{S(V)}} = \frac{1}{2}(\langle{}^3\text{H}|(\boldsymbol{M}^{(1)}+\boldsymbol{M}^{(2)})_z|{}^3\text{H}\rangle + (-)\langle{}^3\text{He}|(\boldsymbol{M}^{(1)}+\boldsymbol{M}^{(2)})_z|{}^3\text{He}\rangle)$$

$$= \mu_{\text{S(V)}}^{(1)} + \mu_{\text{S(V)}}^{(2)} \tag{6}$$

with

$$\boldsymbol{M}^{(2)} = \sum_{X=\pi,\ \text{pair},\ \text{N}^*,\ \omega,\rho} \boldsymbol{M}_X^{(2)}\ . \tag{7}$$

The single particle values are

$$\mu_\text{S}^{(1)} = \frac{1}{2}(\mu_\text{p}+\mu_\text{n})\,(P(\text{S})+P(\text{S}')-P(\text{D})) + \frac{1}{2}P(\text{D}) = 0.408\ \text{n.m.}\ , \tag{8}$$

$$\mu_\text{V}^{(1)} = \frac{1}{2}(\mu_\text{p}-\mu_\text{n})\,(P(\text{S}) - \frac{1}{3}P(\text{S}') + \frac{1}{3}P(\text{D})) - \frac{1}{6}P(\text{D}) = 2.152\ \text{n.m.}\ , \tag{9}$$

where

$$P(\text{S}),\ P(\text{S}')\ \text{and}\ P(\text{D})$$

are respectively the fraction of ${}^2\text{S}_{\frac{1}{2}}$, ${}^2\text{S}'_{\frac{1}{2}}$ and ${}^4\text{D}_{\frac{1}{2}}$ component in the trinucleon bound-state wave function. Using the experimental values of μ_S and μ_V,

$$\mu_\text{S} = 0.42539\ \text{n.m.}\ ,$$

$$\mu_\text{V} = 2.55354\ \text{n.m.}\ , \tag{10}$$

we obtain the discrepancies

$$\delta\mu_\text{S} = \mu_\text{S}^{(\text{exp})} - \mu_\text{S}^{(1)} = 0.017\ \text{n.m.}\ ,$$

$$\delta\mu_\text{V} = \mu_\text{V}^{(\text{exp})} - \mu_\text{V}^{(1)} = 0.401\ \text{n.m.} \tag{11}$$

In Tables I and II, we give the contributions to $\mu_\text{V}^{(2)}$ and $\mu_\text{S}^{(2)}$ associated with various terms in $\boldsymbol{M}^{(2)}$ and components of the trinucleon bound-state wave function. The agreement between $\mu_\text{V}^{(2)}$ and $\delta\mu_\text{V}$ is excellent.

The crucial importance of $\mu_\text{V}^{(2)}$ (N*, SD) and $\mu_\text{V}^{(2)}$ (pair + pionic, SD) was previously noted by one of us (M. R.)[13] in a calculation (based on the simple model of Riska and Brown[14]) which gave $\mu_\text{V}^{(2)} \approx 0.4$ n.m.† The calculated value of $\mu_\text{S}^{(2)}$ is satisfactory in view of our neglect of heavy-meson-exchange, and model-dependent recoil and normalization contributions. Although it is plausible that these neglected terms could account for the remaining small discrepancy in μ_S, they are extremely difficult to calculate because of present uncertainties in the relevant vertex form factors, etc.[8] The heavy-meson exchange contributions calculated in ref. 8 have essentially the same magnitude as the vertex correction of the type represented by

†Riska and Brown[15] also report a rough calculation which yields similar results.

Fig. 1(f). However they are only part of the total heavy-meson-exchange corrections, and there is no reason why the remainder, so far uncalculable, should be small. For consistency, we are thus not including any heavy-meson-exchange contributions.

Padgett, Frank and Brennan[16)] have estimated the contributions of three-body two-pion-exchange currents to μ_V. With simple wave functions they found contributions varying between 0.07 n.m. (for a hard-core radius $r_c = 0.5$f) to 0.15 n.m. (for $r_c = 0.3$f). We are now calculating these three-body contributions for a number of different nucleon-nucleon interactions. In the case of the boundary condition model[17)] with a core radius of 0.72f, the three-body contributions are expected to be negligible.

With our calculated values for μ_S and μ_V, we find $\mu(^3\mathrm{H}) = 2.989$ n.m. and $\mu(^3\mathrm{He}) = -2.163$ n.m. compared to the experimental values 2.97893 n.m. and -2.12815 n.m. respectively. Thus, the single particle and two-nucleon one-pion-exchange contributions alone appear to account for the observed trinucleon magnetic moments to about 1.6%.

In a recent paper,[18)] Gerstenberger and Nogami (GN) have considered a meson-exchange-correction to μ_V in which a proton makes a virtual transition to a neutron (which then interacts via the potential V with another nucleon) and a π^+, the π^+ interacts with the electromagnetic field and is then absorbed by the neutron. If V is the one-pion-exchange potential, then the GN correction is just one of the many renormalization terms for the process $\mathrm{N} + \gamma \rightarrow \mathrm{N} + \pi$ which are already included in our vertex correction terms (Fig. 1(d), (e)). Therefore the GN procedure of adding their correction to that of ref. 8 is not consistent.

A calculation,[19)] similar to the one reported here, has been done for the one-pion-exchange contribution to the axial-vector matrix element for triton beta decay. The dominant contribution to $\delta(\equiv \langle ^3\mathrm{He}|A_\beta^{(2)}|^3\mathrm{H}\rangle/\langle ^3\mathrm{He}|A_\beta^{(1)}|^3\mathrm{H}\rangle)$ is the S-D matrix element of the meson-exchange current involving the (N_{33}^*–N–axial current) vertex. This term alone contributes about 12.2%* to δ whereas $\delta^{\mathrm{exp}} \approx 6\%$. The total calculated δ is 14.8%. These results are in qualitative agreement with those of Riska and Brown[14)] who find $\delta_{\mathrm{SD}}(\mathrm{N}_{33}^*) \approx 8.8\%$ and $\delta_{\mathrm{Tot}} \approx 12.7\%$. One may wonder why the two-nucleon current gives good results for trinucleon magnetic moments and unsatisfactory results for δ in triton beta decay. The only essential difference in the calculations, as far as the N_{33}^* vertex correction is concerned, is in the (N_{33}^*–N–current) couplings. The estimate of the (N_{33}^*–N–axial current) coupling, based on PCAC, may be too large or perhaps there is something fundamentally incorrect in the basic formalism used in calculating meson-exchange correction.

Table I. Contributions to $\mu_V^{(2)}$ associated with various $\mathrm{M}^{(2)}$ (X) and components of the trinucleon wave function.

X	$\mu_V^{(2)}(X, \mathrm{SS})$	$\mu_V^{(2)}(X, \mathrm{S'S'})$	$\mu_V^{(2)}(X, \mathrm{SD})$	Total (n.m.)
N*	0.002	−0.000	0.164	0.166
ω	0.000	−0.000	0.012	0.012
Pair & Pionic	0.138	−0.002	0.105	0.241
Total	0.140	−0.002	0.281	0.419

* Several errors in the formulas used in ref. 19 have been corrected in obtaining these results. A detailed discussion of this calculation will be given elsewhere.

Table II. Contributions to $\mu_S^{(2)}$ associated with $\boldsymbol{M}^{(2)}(\rho)$ and various components of the trinucleon wave function.

X	$\mu_S^{(2)}(X, \mathrm{SS})$	$\mu_S^{(2)}(X, \mathrm{S'S'})$	$\mu_S^{(2)}(X, \mathrm{SD})$	Total (n.m.)
ρ	-0.000	-0.000	0.010	0.010

References

1) E. P. Harper, Y. E. Kim and A. Tubis: Phys. Rev. Letters **28** (1972) 1533.
2) L. D. Faddeev: Zn. Eksper. teor. Fiz. **39** (1960) 1459 [trans.: Soviet Physics-JETP **12** (1961) 1014].
3) R. A. Malfliet and J. A. Tjon: Ann. Phys. (N.Y.) **61** (1970) 425.
4) E. P. Harper, Y. E. Kim and A. Tubis: Phys. Rev. **C2** (1970) 877; **C2** (1970) 2455 (E).
5) E. P. Harper, Y. E. Kim and A. Tubis: Phys. Rev. **C6** (1972) 126.
6) R. V. Reid: Ann. Phys. (N.Y.) **50** (1968) 411.
7) C. Gignoux and A. Laverene: Phys. Rev. Letters **29** (1972) 436.
8) M. Chemtob and M. Rho: Nuclear Phys. **A163** (1971) 1.
9) F. Villars: Helv. Phys. Acta **20** (1947) 476.
10) G. F. Chew, M. L. Goldberger, F. E. Low and Y. Nambu: Phys. Rev. **106** (1957) 1345.
11) R. E. Cutkosky: Phys. Rev. **112** (1958) 1027.
12) H. D. Young and R. E. Cutkosky: Phys. Rev. **117** (1960) 595.
13) M. Rho: Nuclear Structure Symposium of the Thousand Lakes, Joutsenlampi, Finland (1970).
14) D. O. Riska and G. E. Brown: Phys. Letters **32B** (1970) 662.
15) D. O. Riska and G. E. Brown: Phys. Letters **32B** (1972) 195.
16) D. W. Padgett, W. M. Frank and J. G. Brennan: Nuclear Phys. **73** (1965) 424, 445.
17) H. Feshbach and E. L. Lomon: Ann. Phys. (N.Y.) **48** (1968) 94.
18) R. V. Gerstenberger and Y. Nogami: Phys. Rev. Letters **29** (1972) 233.
19) E. Fischbach, E. P. Harper, Y. E. Kim, A. Tubis and W. K. Cheng: Phys. Letters **38B** (1972) 8.

Discussion

H. MIYAZAWA (Univ. of Tokyo): For the three nucleon system you got a rather large value of $\delta\mu_v = 0.4$ from the meson effect. Could you guess how much $\delta\mu_v$ you would get for nuclei of the Ca region? I'm asking this because Sugimoto and others obtained a large $\delta\mu_v$ of about 0.5.

KIM: I have not calculated $\delta\mu_v$ for such nuclei, so I could not guess.