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VI.i. Relativistic Correction to Nuclear Magnetic Moment

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(Presented by H. Ohtsubo)

Recent experimental progress is very rapid, and precision of experimental data becomes higher and higher. In this connection, we have to calculate the magnetic moment with a higher accuracy. One of these corrections is a group called exchange currents. They consist of the meson exchange currents, isobar effects, and relativistic corrections. These have been studied by many authors to different extents. In particular, the relativistic corrections were examined by Margenau and Caldirola thirty years ago[1,2] for a restricted case.

We report about the treatment of the relativistic corrections to the nuclear magnetic moment as well as to the Gamow-Teller matrix element for beta decay in the general way.

The motion of a nucleon inside the nucleus can be described by the Dirac equation with the effective potential $V(\boldsymbol{r})$ between nucleon and the residual nucleus.

$$[\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta M + V(\boldsymbol{r})]\,\Psi(\boldsymbol{r}) = E\Psi(\boldsymbol{r}) \ .$$

If, for simplicity, the effective potential is assumed to be the fourth component of the vector, the single nucleon wave function is given by a four-component spinor

$$\Psi_{j,l}(\boldsymbol{r}) = \begin{pmatrix} g_{j,l}(r)\chi_{j,l}(\hat{r}) \\ if_{j,l}(r)\chi_{j,\bar{l}}(\hat{r}) \end{pmatrix}, \ l = j \pm 1/2, \bar{l} = j \mp 1/2,$$

where $\chi_{j,l}$ is angular momentum wave function of two-component, and the subscripts l and j represent the orbital and total angular momenta of a single particle, respectively. The normalization of the wave function is taken as

$$\int_0^\infty r^2 \mathrm{d}r[g_{j,l}^2(r) + f_{j,l}^2(r)] = 1.$$

The small component $f_{j,l}(r)$ represents the effect of relativistic motion of the nucleon. With this wave function we calculate the magnetic moment of a single particle state j. Operator of magnetic moment $\hat{\mu}$ is given by the equation

$$\hat{\mu}(\boldsymbol{r}) = M\gamma_0 \boldsymbol{r}\times\boldsymbol{\alpha} + \gamma_a \beta\boldsymbol{\sigma} \ ,$$

where γ_0 and γ_a are the Dirac and anomalous magnetic moments of the nucleon, respectively. The result is as follows:

$$\mu = \mu_s + \delta\mu \ ,$$

$$\delta\mu = -\left[\frac{(2j+1)^2}{2(j+1)}\gamma_0 + \frac{1}{(j+1)}\gamma_a\right]\varepsilon \ .$$

μ_s is the Schmidt value. $\delta\mu$ is the relativistic correction to nuclear magnetic moment. ε is given by the integral

$$\varepsilon = \int_0^\infty r^2 \mathrm{d}r f_{j,l}^2(r) .$$

To estimate the correction $\delta\mu$, we take simply the square well for the effective potential. The well depth is adjusted so as to give the experimental separation energy of the nucleon. Performing numerical calculations, we have the magnitude of ε from 0.01 to 0.02, which is almost independent of the nuclear states. It should be noticed that the correction depends upon j, but not l, so that we cannot express the correction in terms of the effective g-factors, g_s and g_l. The numerical values of the correction $\delta\mu$ for various nuclei are given in the table below. For example, the corrections are, in units of nuclear magneton, $-.26$ for nitrogen 15, $+.013$ for oxygen 15, $+.009$ for oxygen 17, and $-.093$ for fluorine 17, respectively. In the region of lead 208, the correction for proton orbit reaches up to -0.2, while it remains to be small for neutron orbit. As is expected, the shifts from the Schmidt value due to the correction are downwards for proton, and upwards for neutron. It is small for higher angular momentum j for neutron, but it is large for proton.

A similar correction to the Gamow-Teller matrix element should be considered in the case of beta decay. The Gamow-Teller matrix element between mirror nuclei is given as

$$\begin{aligned} \int \sigma &= \left(\int \sigma\right)_{\text{N.R.}} + \delta\left(\int \sigma\right) \\ &= \left(\int \sigma\right)_{\text{N.R.}} \left[1 - \frac{2j+1}{j+1}\varepsilon\right], j = l + 1/2, \\ &= \left(\int \sigma\right)_{\text{N.R.}} \left[1 - \frac{2j+1}{j}\varepsilon\right], j = l - 1/2. \end{aligned}$$

The first term in the right-hand side of the first equation represents the usual Gamow-Teller matrix element, and the second the correction. The correction is, in contrast to the magnetic moment, dependent upon both j and l. Even in the high angular momentum state, the correction remains to be finite, as long as the integral ε is almost independent of the nuclear states. Here we give the numerical values of our corrections for typical examples of the beta decay between mirror nuclei. We can see from the above results that we cannot neglect the relativistic corrections for the magnetic moment and the Gamow-Teller matrix element of beta decay.

To see more clearly what we have done, we take another approach to the relativistic corrections. We diagonalize the Hamiltonian H under unitary transformation and transform a four-component wave function Ψ into a two-component wave function Φ, which we usually regard as the non-relativistic wave function. The new Hamiltonian $\bar{H}$ and the wave function Φ are related to the old ones by unitary operator S.

$$H\Psi = E\Psi \to \bar{H}\Phi = E\Phi ,$$

$$\bar{H} = SHS,^\dagger \; \Phi = S\Psi = \begin{pmatrix} \phi \\ 0 \end{pmatrix} .$$

The expectation value of any operator Ξ is given by

$$\begin{aligned}\langle\Psi, \Xi\Psi\rangle &= \langle\Phi, S\Xi S^{\dagger}\Phi\rangle \\ &= \langle\phi, \Xi_{\mathrm{eff}}\Phi\rangle.\end{aligned}$$

That is, the expectation value is represented by the two-component wave function and the effective operator Ξ_{eff}. The procedure to obtain the effective operator is known by the name of Foldy transformation.

$$\begin{aligned}&H = \boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta M + \Omega_{\mathrm{e}} + \Omega_{\mathrm{o}} \\ &\rightarrow \bar{H} = \beta M + \Omega_{\mathrm{e}} + \frac{1}{2M}\beta(\boldsymbol{\alpha}\cdot\boldsymbol{p} + \Omega_{\mathrm{o}})^2 \qquad \Omega_{\mathrm{e}}\text{: Even operator,} \\ &\quad - \frac{1}{8M^2}[\boldsymbol{\alpha}\cdot\boldsymbol{p} + \Omega_{\mathrm{o}}, [\boldsymbol{\alpha}\cdot\boldsymbol{p} + \Omega_{\mathrm{o}}, \Omega_{\mathrm{e}}]_-]_- + \ldots, \quad \Omega_{\mathrm{o}}\text{: Odd operator.}\end{aligned}$$

The interaction part of the total Hamiltonian H usually involves the even and odd operators Ω_{e} and Ω_{o}. These operators may include the elctromagnetic or weak interaction, in addition to the strong interaction. The relativistic correction in our study arises from the fourth term in the right-hand side of the new Hamiltonian. This term consists of the velocity-dependent one-body operators and the operators related to the nuclear forces. In principle, the above two approaches are equivalent to each other. For the purpose of the numerical calculations of the magnetic moment or the Gamow-Teller matrix element, we must obtain the effective operator Ξ_{eff} by a complicated procedure in the second approach. In the first approach, we can perform the calculation if we know the relativistic wave function. Therefore, the first approach is more suitable for the present purpose.

The assumption of the scalar potential $\beta V(r)$ in the Dirac equation was also investigated.

Table I. ^{88}Sr, etc., should read the region of ^{88}Sr, etc.

Proton orbit						Neutron orbit					
$A=11$	$1p_{3/2}$	-0.050	$A=35$	$1d_{3/2}$	-0.049	$A=11$	$1p_{3/2}$	$+0.010$	$A=35$	$1d_{3/2}$	$+0.010$
13	$1p_{1/2}$	-0.024	37	$1d_{3/2}$	-0.045	13	$1p_{1/2}$	$+0.012$	37	$1d_{3/2}$	$+0.009$
15	$1p_{1/2}$	-0.026	39	$1d_{3/2}$	-0.046	15	$1p_{1/2}$	$+0.013$	39	$1d_{3/2}$	$+0.009$
17	$1d_{5/2}$	-0.093	41	$1f_{7/2}$	-0.130	17	$1d_{5/2}$	$+0.009$	41	$1f_{7/2}$	$+0.007$
19	$2s_{1/2}$	-0.047	43	$1f_{7/2}$	-0.131	19	$2s_{1/2}$	$+0.024$	43	$1f_{7/2}$	$+0.007$
21	$1d_{3/2}$	-0.059	^{88}Sr	$2p_{1/2}$	-0.032	21	$1d_{3/2}$	$+0.011$	^{114}Sn	$3s_{1/2}$	$+0.021$
23	$1d_{3/2}$	-0.061	^{88}Sr	$2p_{3/2}$	-0.047	23	$1d_{3/2}$	$+0.012$	^{114}Sn	$2d_{3/2}$	$+0.012$
25	$1d_{5/2}$	-0.080	^{88}Sr	$1g_{9/2}$	-0.148	25	$1d_{5/2}$	$+0.008$	^{114}Sn	$1h_{11/2}$	$+0.005$
27	$1d_{5/2}$	-0.083	^{208}Pb	$1h_{9/2}$	-0.115	27	$1d_{5/2}$	$+0.008$	^{208}Pb	$3p_{1/2}$	$+0.020$
29	$2s_{1/2}$	-0.036	^{208}Pb	$1i_{13/2}$	-0.213	29	$2s_{1/2}$	$+0.018$	^{208}Pb	$1i_{13/2}$	$+0.004$
31	$2s_{1/2}$	-0.038				31	$2s_{1/2}$	$+0.019$	^{208}Pb	$2f_{5/2}$	$+0.008$
33	$1d_{3/2}$	-0.047				33	$1d_{3/2}$	$+0.009$	Pb	$2g_{9/2}$	$+0.007$

References

1) H. Margenau: Phys. Rev. **57** (1940) 383.
2) P. Caldirola: Phys. Rev. **69** (1946) 608.

Discussion

H. MIYAZAWA (Univ. of Tokyo): Are there any big differences in the results for the scalar and vector potentials? To get a large spin-orbit coupling the vector potential is favorable.

OHTSUBO: The numerical table gives the relativistic corrections for the vector potential. For the scalar potential, the numerical values are almost the same for the neutron, but may change sign and magnitude for the proton.

G. E. BROWN (Stony Brook): I would like to amplify these remarks. There is a large assumption made, that a relativistic many-body problem is simplified to a single-particle problem with a central scalar potential in a sort of a Hartree-Fock description. Now, that is probably all right in calculating the Pauli part of the moment, because in that case one just gets the Lorentz contraction to the moment; the motion simply contracts the moment. However, Breit showed in the late 1940's in the case of the deuteron that the reduction of moment coming from the $\boldsymbol{\alpha} \times \boldsymbol{r}$ is extremely model dependent and depends upon the interaction between the particles. Therefore, for the Dirac moment I don't think one can trust your results.

OHTSUBO: As was pointed out by Prof. Brown, the Dirac moment is very dependent upon the nuclear forces. We should do the complete calculation in the following way. First, we calculated a self-consistent wave function with realistic nuclear forces, *e.g.* a one-boson-exchange potential, and then estimate the correction. Then there would be no ambiguity. Dr. Yazaki has made a similar calculation. In the case of the neutron the correction of the magnetic moment may not be altered so much, because the correction depends on the probability of small components involved in the wave function. In the case of the Gamow-Teller matrix elements, the situation is the same as for the neutron.

D. H. WILKINSON (Oxford Univ.): How did you determine the radius which you used?

OHTSUBO: We used the empirical radius.

WILKINSON: I believe that the choice of radius for your square well may be quite important. I do not think that you should choose the radius to be equal to the size of the nucleus but rather of such a size that you fit not only the binding energy of the last nucleon but also the r.m.s. size of its orbital as given, for example, by a "realistic" Saxon-Woods potential that reproduces the overall charge distribution. This would give to the last nucleon a significantly more extended wave function, particularly in the lighter nuclei, and therefore, presumably, a smaller relativistic correction than you have found.

OHTSUBO: The result depends, in principle, on the radius. But, the effect is not too significant.