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VII.c. Higher Nuclear Deformations from the Bound-Muonic Point of View

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(Presented by H. Narumi)

In muonic hyperfine spectra recently observed in the rare earth and actinide regions it is possible to study the nuclear charge deformation by analyzing the mixing of the muonic states with the low-lying nuclear rotational states through the electromagnetic interaction.[1-4] From such studies one can obtain not only a more reasonable form of the nuclear charge distribution but also other information about nuclear structure such as the intrinsic quadrupole moment. In previous analyses of muonic hyperfine spectra only the nuclear quadrupole deformation has been considered and higher multipole interactions of the muon with the nucleus have not been taken into account. However, higher order components in the nuclear deformation have now been examined in recent experiments on α scattering and Coulomb excitation.[5-7] In fact, it is expected that for some nuclei the effect of the Y_4 deformation or the electric hexadecapole (E4) interaction on muonic spectra may not always be negligible in the analysis of precise measurements of the muonic X-rays.

In this paper we consider the contribution of the Y_4 deformation to the transition matrix elements of the muon-nucleus system and re-evaluate the intrinsic quadrupole moment for certain nuclei, such as ^{152}Sm and ^{184}W for which we can expect a large Y_4 deformation parameter β_4.

In our analysis a modified Fermi charge distribution

$$\rho(r) = N[1 + \exp\{[r - c(1 + \sum_l \beta_l Y_{l0}(\theta))]/a\}]^{-1} \tag{1}$$

is generally assumed, where the half-radius c is modified by an angular dependence, a is the surface thickness parameter, and β_l the electric 2^l-pole deformation parameter, where all the parameters are settled with the corrections of quantum-electrodynamics and nuclear polarizations for each state. We can expand $\rho(r)$ into multipole terms:

$$\rho(r) = \rho_0(r) + \sum_l \rho_l(r) Y_{l0}(\theta) . \tag{2}$$

The electric multipole interaction of the muon-nucleus system can be written as follows:

$$H_l = \frac{1}{2} e Q_l f_l(r) P_l(\cos \mathscr{I}), \tag{3}$$

$$f_l(r) = \sqrt{\frac{4\pi}{2l+1}} \frac{2}{Q_l} \left(r^l \int_r^\infty \rho_l(r') r'^{1-l} \mathrm{d}r' + r^{-(l+1)} \int_0^r \rho_l(r') r'^{l+2} \mathrm{d}r' \right), \tag{4}$$

$$Q_l = 2\sqrt{\frac{4\pi}{2l+1}} \int_0^\infty \rho_l(r) r^{l+2} \mathrm{d}r , \tag{5}$$

where Q_l is the intrinsic El moment, and $l = 2$ or 4 indicates the quadrupole or hexadecapole moment, respectively. The angle $\mathscr{I}$ gives the position of the muon with respect to the nuclear symmetry axis and $f_l(r)$ is the form factor of the El interaction.

The unperturbed eigenfunction of the total angular momentum F is denoted by

$$|IK,\, n\kappa j;\, FM\rangle = \sum_m (IM_I,\, jm|FM)|IM_I K\rangle|n\kappa jm\rangle\,, \tag{6}$$

where $|n\kappa jm\rangle$ is an eigenstate of the muon which can be found numerically for the monopole density $\rho_0(r)$. In our calculation $\rho_0(r)$ is replaced by the Fermi-type distribution with known effective parameters.[8] The nuclear eigenfunction $|IMK\rangle$ is given by

$$|IMK\rangle = \frac{1}{4\pi}\sqrt{\frac{2I+1}{1+\delta_{K0}}}\{D^I_{M,K}(\theta,\phi,\psi) + (-1)^I D^I_{M,-K}(\theta,\phi,\psi)\}, \tag{7}$$

for even-even nuclei, where θ, ϕ and ψ are the Eulerian angles of the nuclear axis. The matrix element of the muon-nucleus multipole interaction is obtained by use of the basis function (6):

$$\begin{aligned}&\langle I'K'n'\kappa'j'FM|H_l|IKn\kappa jFM\rangle\\ &= -\frac{e^2Q_l}{10}(-1)^{I'+I-K'+F+\frac{1}{2}}5\sqrt{(2I'+1)(2I+1)(2j'+1)(2j+1)}\\ &\times\begin{Bmatrix}F & j' & I'\\ l & I & j\end{Bmatrix}\begin{pmatrix}I' & l & I\\ -K' & 0 & K\end{pmatrix}\begin{pmatrix}j' & l & j\\ -\frac{1}{2} & 0 & \frac{1}{2}\end{pmatrix}\langle n'\kappa'j'|f_l(r)|n\kappa j\rangle.\end{aligned} \tag{8}$$

The energy levels and the eigenstates of the system are obtained by diagonalizing the matrix of the total Hamiltonian with given n, l and F. The eigenstates are then admixtures of unperturbed states, where the multipole interaction in muonic states above the 3d states is not considered. The muonic hyperfine spectrum can be deduced from these energy levels together with the relative intensities of all possible electric dipole transitions. On the other hand, the quantities depending on the nuclear charge density, as obtained from observed hyperfine spectra, are the unperturbed muonic binding energies and the E2 interaction energies of the muonic 2p and 3d states denoted by

$$\alpha^{(2)}_{n\kappa'\kappa} = -\frac{e^2}{10}Q_2\langle n\kappa'|f_2(r)|n\kappa\rangle \tag{9}$$

for the case without Y_4 deformation. If the Y_4 deformation is included, the E4 interaction energy

$$\alpha^{(4)}_{3d} = -\frac{e^2}{10}Q_4\langle 3d'|f_4(r)|3d\rangle \tag{10}$$

along with the kinematical factor would be added to the E2 interaction energy $\alpha^{(2)}_{3d}$ in the matrix element for the 3d state. In addition the matrix elements also depend on β_4 through the nuclear charge density.

Thus we can discuss the influence of the Y_4 deformation in comparing theoretical with

Table I. Values of the intrinsic quadrupole moment (in barns).*

	^{152}Sm	^{184}W
$Q_2^{(\mu)}$	5.78	6.27
	(0.538; 0.296)	(0.493; 0.237)
$Q_2^{(\mu)}(\beta_4)$	5.84	6.16
	(0.540; 0.278)	(0.480; 0.251)
$Q_2^{(c)}$	5.85 ± 0.13	6.21 ± 0.06

* The former and renormalized values of a(fm) and β_2 are written in parentheses; the other charge parameter c is invariant under renormalization.

observed values of the charge distributions and the E2 moments obtained from muonic spectra.[4] For some fixed values of the unperturbed energy of the muon and of the nuclear E2 moment, the Y_4 effect changes the values of $\alpha_{2p}^{(2)}$ and $\alpha_{3d}^{(2)}$ effectively by -1.3 per cent and about 7 per cent for ^{152}Sm, and $+2.4$ per cent and about 5 per cent for ^{184}W, if values of β_4 are assumed to be $+0.09$[6] and -0.08,[9] respectively. In accordance with these changes the charge distribution or the intrinsic E2 moment can be re-evaluated by renormalizing the matrix elements of the 2p and 3d states. The results are shown in Table I, where the former values[4] of $Q_2^{(\mu)}$ and the renormalized ones $Q_2^{(\mu)}$ (β_4) for the E2 moment are compared with values of $Q_2^{(c)}$ obtained from Coulomb excitation experiments which are model independent.[10,11] When the Y_4 deformation is taken into account it is noted that the value of the E2 moment increases or decreases according to the sign of β_4.

In conclusion we have found that corrections considered in this paper are of the order of magnitude of the experimental errors. However, the analysis of the Y_4 deformation from the muonic point of view might be more in line with the accuracy obtainable in muonic and other model independent experiments.

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Discussion

H. J. LEISI (ETH, Zürich): I would like to ask why the effect in $\alpha_{2p}^{(2)}$ is smaller than the effect in $\alpha_{3d}^{(2)}$. I would expect it to be the other way around.

NARUMI: The percentage deviation from β_4 in $\alpha_{3d}^{(2)}$ is larger than that in $\alpha_{2p}^{(2)}$, but the absolute value of $\alpha_{3d}^{(2)}$ is one order of magnitude smaller than that of $\alpha_{2p}^{(2)}$.