

THE SCREENED ELECTRON PHONON INTERACTION
IN DEGENERATE SEMICONDUCTORS

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In this paper, we present new calculations of screening which include a) non-local effects, b) exchange, c) correlation and d) vertex corrections. These latter corrections, traditionally neglected in semiconductor physics are found to have a significant effect on the mobility, the carrier temperature as well as a direct effect on the screened phonon generation rate recently measured by Narayanamurti, et al.

Recent experimental results of Narayanamurti, Logan, Chin and Lax [1,2] on phonon generation in GaAs have demonstrated a sensitivity to the phonon type and propagation direction, the lattice temperature, the applied field or electron temperature, a direct effect of carrier concentration and an indirect effect via screening. Since many of these parameters can be varied experimentally, we have an ideal tool for studying phonons, electron-phonon interactions and screening in solids.

The piezoelectric electron-phonon interaction is usually quite difficult to observe in transport processes because deformation potential interactions usually dominate near room temperature. By the time the temperature is lowered enough to cause piezoelectric interactions to exceed deformation potential interactions, impurity scattering dominates them both. This statement is true, even for the measurements reported here, but since the piezoelectric phonons are observed directly we have a unique opportunity to compare the strengths of the piezoelectric and deformation potential interactions unhindered by impurity scattering.

Since carrier concentrations of $10^{16}/\text{cm}^3$ at a few degrees Kelvin in GaAs yields excellent metallic behavior:

- 1) Screening must be applied to the electron-phonon interaction and not just to the impurity scattering process, as is customarily done.
- 2) Since the rate of phonon production is quite sensitive to screening we have a) a need for a careful examination of the screening process and b) an opportunity to make a direct study of the screening process.

We shall state briefly, without proof, the rates of phonon production by deformation potential and piezoelectric scattering processes. We shall see that these rates are proportional to the excess $T_e - T_p$ of the electron temperature to the phonon or lattice temperature. The latter, in turn, is determined using an energy balance condition involving the applied field and the electron mobility. The latter in turn is dominated by impurity scattering. The chief

purpose for stating these formulas is to demonstrate their sensitivity to screening.

For simplicity we shall only state formulas here appropriate to the degenerate limit. The phonon power per carrier P/N by deformation potential scattering is given by

$$(P/N)_{\text{def}} = \frac{3}{2\pi^2} \frac{T_e - T_p}{T_B} \frac{m^*}{\rho a_B^3} \frac{|E_1|^2}{\hbar} y f(y). \quad (1)$$

The corresponding formula for piezoelectric scattering is

$$(P/N)_{\text{pe}} = \frac{3}{2\pi^2} \frac{T_e - T_p}{T_B} \frac{m^*}{\rho a_B^3} \frac{e^2 h_{14}^2}{2\hbar k_f^2} y g(y) \hat{R}, \quad (2)$$

and the mobility is given by $\mu = e\tau/(m^*)$ where the ionized impurity scattering rate is given by

$$\frac{1}{\tau} = \frac{8}{3\pi} \frac{n_I}{n} \frac{E_B}{\hbar} f_{-1}(y). \quad (3)$$

In these expressions, E_1 is the deformation potential, $h_{14} = e_{14}/\epsilon_b$ where ϵ_b is the background dielectric constant and $e_{14} = e_{123}$ is the piezoelectric constant relating polarization to strain. The Bohr radius a_B associated with a donor in GaAs is 103 Å and the donor binding energy $E_B = kT_B$ with $T_B = 62.6$ K. With an effective mass $m^* = 0.06$ of the free electron mass, and a density $\rho = 5.378$ g/cm³, the dimensionless ratio $m^*/(\rho a_B^3) = 1.02 \times 10^{-11}$. \hat{R} is a dimensionless factor that contains selection rule and finite aperture corrections described in more detail elsewhere. The ionized donor concentration is n_I and the free electron concentration is n . [In the absence of compensation $n_I/n = 1$].

The Fermi wave-number is denoted k_f , and y is the dimensionless parameter, $y = \pi k_f a_B = 6.03/r_s$ where $r_s = [3/(4\pi n)]^{1/3}/a_B$ is the metal physicists parameter which is in the typical range $2 < r_s < 6$ for real metals. For GaAs at $n = 10^{16}$ /cm³, $r_s = 2.78$ - a good metal!

All of the difficulties of the calculation are concealed in the screening factors $f(y)$, $g(y)$ and $f_{-1}(y)$ which are discussed below. The temperature rise is determined by equating the power gain per electron to the power loss: $e\mu F^2 = (P/N)_{\text{def}} + (P/N)_{\text{pe}}$ where F is the applied field. We find that the temperature rise is given by

$$T_e - T_p = T_B \frac{\pi^3}{4} \frac{\rho a_B^3}{m^* n_I} \frac{n}{y f(y) f_{-1}(y)} \frac{1}{1 + (4/5)R} \frac{F^2}{(E_1/e)^2} \frac{\hbar}{m^* \omega_B} \quad (4)$$

where \hat{R} has been given an average value for all orientations of $4/5$ and

$$R = \frac{P_{\text{pe}}}{P_{\text{def}}} = \frac{h_{14}^2}{2(E_1/e)^2 k_f^2} \frac{g(y)}{f(y)} \quad (5)$$

is the ratio of piezoelectric to deformation potential powers.

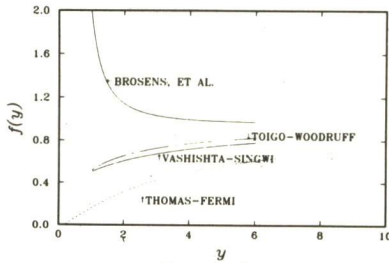


Fig. 1

The screening factor $f(y)$ for deformation potential scattering

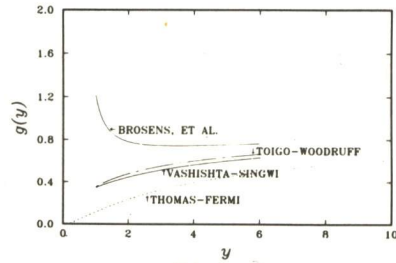


Fig. 2

The screening factor $g(y)$ for piezoelectric scattering

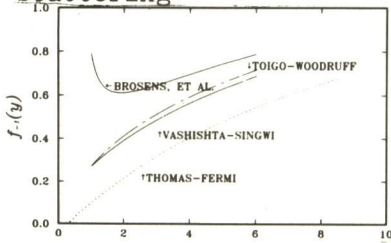


Fig. 3

The screening factor $f_{-1}(y)$ for impurity scattering

The screening functions $f(y)$ and $g(y)$ have been defined so that they approach unity as $y \rightarrow \infty$ thus leading to the correct limit when screening and vertex corrections are absent. With this normalization $f(y) = 4 f_3(y)$, $g(y) = 2 f_1(y)$ where all $f_n(y)$ are given by

$$f_n(y) = \int_0^1 \left[\frac{u^2 \Lambda_v(u)}{u^2 + L(u) \Lambda(u)/y} \right]^2 u^n du. \quad (6)$$

The integration variable arose from the transformation $u = q/2k_F$. The integrand describes screening of a charge distribution of wave-vector k . The exponents $n = 3, 1$ and -1 describe the relative importance of different wave-vectors for the deformation potential case, the piezoelectric scattering case, and impurity scattering respectively.

The wave-vector dependence of the screening in the free-electron limit is described by the Lindhard function

$$L(u) = \frac{1}{2} + \frac{1}{4} \frac{1-u^2}{u} \ln \left| \frac{1+u}{1-u} \right|. \quad (7)$$

The vertex correction $\Lambda_v(u)$ is frequently ignored (set equal to unity). However, as Kukkonen and Wilkens [3] have shown for small q , and we have shown for all q , it is a good approximation to set $\Lambda_v(u) \approx \Lambda(u)$ where $\Lambda(u)$ is the enhancement of the proper polarization due to exchange and correlation effects. This enhancement can be written in the form

$$\Lambda(u) = \left[1 - \frac{G(u) L(u)}{y u^2} \right]^{-1}, \quad (8)$$

where $G(u)$ is a measure of the exchange correlation potential $V_{xc}(q)$ induced by a charge density fluctuation $\delta n(q)$ induced at wave-vector q .

Calculations of $G(u)$ have been made by Toigo and Woodruff [4], Vashishta and Singwi [5] and by Brosens, DeVreese and Lemmons [6]. The influence that these different screening approximations make on our screening functions is demonstrated in Figs. 1-3 where they are compared with each other and the Thomas-Fermi approximation. Although the Vashishta-Singwi and Toigo-Woodruff procedures are quite different, their results are indistinguishable experimentally. Our experimental data do not seem to support the Brosens' rapid variations, but a careful analysis of the data is not yet completed and will be published elsewhere.

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