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3.53 Spin Polarization Effects in the $^3H(d,n)^4He$ Fusion Reaction

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A recent investigation has shown that the $^3H(d,n)^4He$ fusion reaction rate could be enhanced by a factor of 3/2 if the fusion plasma consisted of both polarized deuterons and tritons, forming exclusively the channel-spin $S = 3/2$, $J = 3/2^+$ state[1]. This result follows simply from the statistical weights of the quartet $S = 3/2$ and doublet $S = 1/2$ initial states, with the assumption of the single $J = 3/2^+$ reaction amplitude.

Since, with a small but nonzero $J = 1/2^+$ amplitude, the maximum enhancement of the reaction occurs at the peak of the $J = 3/2^+$ resonance, corresponding to a deuteron lab energy of 107 keV, it is of obvious interest to know what the enhancement would be at the lower energies that are typical of fusion plasmas. We are able to address this question by extending earlier calculations which gave the values of all of the spin-polarization observables at this $J = 3/2^+$ resonance in both the $^3H(d,n)^4He$ and the $^3He(d,p)^4He$ reactions[2].

With the inclusion of a $J = 1/2^+$ amplitude, for which $l = 0$, $s = 1/2$, $l' = 0$, $s' = 1/2$, two additional K-matrix elements appear in eq.(4) of reference 2. These are, for $J = 1/2^+$,

$$K_{1/2\ 1/2} = K_{-\ 1/2\ -\ 1/2} = \frac{\sqrt{\pi}}{k}\, Y_0(\theta)U(\tfrac{1}{2}) \quad , \qquad (1)$$

where U(J) is the reaction amplitude for the state of total angular momentum J. Defining

$$U(\tfrac{1}{2})/U(\tfrac{3}{2}) \equiv r\, e^{i\delta} \quad , \qquad r = |U(\tfrac{1}{2})|/|U(\tfrac{3}{2})| \qquad (2)$$

is the ratio of the absolute values of the $J = 1/2$ and $J = 3/2$ reaction amplitudes. The M-matrix elements (eq.(5)), reference 2) then become

$$\begin{aligned}
M_{1/2,1\ 1/2} &= \sqrt{1/5}\ Y_2^1(\theta)U &&= (-iA - D)/\sqrt{2} \\
M_{1/2,0\ 1/2} &= [\sqrt{4/15}\ Y_2^0(\theta) - \sqrt{1/3}\ Y_0 r\, e^{i\delta}]U &&= F \\
M_{1/2,-1\ 1/2} &= \sqrt{1/5}\ Y_2^{-1}(\theta)U &&= (-iA + D)/\sqrt{2} \\
M_{1/2,1\ -\ 1/2} &= [\sqrt{2/15}\ Y_2^0(\theta) + \sqrt{2/3}\ Y_0 r\, e^{i\delta}]U &&= (-B - C)/\sqrt{2} \\
M_{1/2,0\ -\ 1/2} &= \sqrt{2/5}\ Y_2^{-1}(\theta)U &&= E \\
M_{1/2,-1\ -\ 1/2} &= \sqrt{4/5}\ Y_2^{-2}(\theta)U &&= (-B + C)/\sqrt{2}
\end{aligned} \qquad (3)$$

with $U = (i\sqrt{\pi}/k)U(\tfrac{3}{2})$.

From eq.(3) we then find that

$$\begin{aligned}
A &= 0 & D &= E = N(3\sin\theta\cos\theta) \\
B &= -N(1 + r\, e^{i\delta}) & F &= N[(3\cos^2\theta - 1) - r\, e^{i\delta}] \quad , \\
C &= -N[(3\cos^2\theta - 2) + r\, e^{i\delta}] & &\text{with } N = U/2\sqrt{3}
\end{aligned} \qquad (4)$$

Eqs.(4) reduce to eqs.(6) of reference 2 for $r = 0$.

From the tabulation[3,4] of polarization observables in terms of the amplitudes A to F, we have the following results:

a) $\sigma(\theta) = \frac{2}{3}\,\sigma(\tfrac{3}{2})\left(1 + \frac{r^2}{2}\right)$, where $\sigma(\tfrac{3}{2})$ is the differential cross section with only the $J = 3/2^+$ state contributing. Since the cross-section with both deuterons and tritons polarized to form the quartet S = 3/2 state is equal to $\sigma(3/2)$, we define an enhancement factor

$$f \equiv \sigma(\tfrac{3}{2})/\sigma = \frac{3}{2}\left(1 + \frac{r^2}{2}\right)^{-1} \quad . \tag{5}$$

One sees that the spin-polarization enhancement of the cross section is related directly to the ratio of the reaction amplitudes, eq.(2). Thus, for $r = 0$, $f = 3/2$ and for $r = 1$, $f = 1$, i.e. no enhancement. Clearly, the maximum enhancement of the reaction rate occurs at the peak of the $J = 3/2^+$ resonance since r has a minimal value there. Then since r increases with decreasing energy below E_d = 107 keV, the enhancement of the reaction rate decreases correspondingly. Assuming that U(1/2,E) is essentially constant below E_d = 107 keV, the energy dependence of r^2 can be estimated from

$$r^2(E) = \frac{\left|U(\frac{1}{2},E)\right|^2}{\left|U(\frac{3}{2},E)\right|^2} = C\left[(E - E_R)^2 + \frac{\Gamma^2}{4}\right] \quad , \tag{6}$$

with Γ_{CM} = 80 ± (16-20) keV for the J = 3/2 resonance. Then with the estimate[1] that $(2/3)f(E) = 0.95$ at E_R = 64 keV (E_d = 107 kev), the energy dependence of f(E) is shown in Table I.

Table I. Energy dependence of the spin-polarization enhancement factor f(E) in the $^3H(d,n)^4He$ fusion reaction

E_d(keV)	E_{CM}	$(E - E_R)^2$	$r^2(E)$	f(E)
107	64	0	0.105	1.43
73	44	$(\Gamma/4)^2$	0.131	1.41
40	24	$(\Gamma/2)^2$	0.210	1.35

It is seen that at E_d = 40 keV f(E) = 1.35, which is an appreciable reduction from the maximum possible value of 1.5. This 40 keV lab energy corresponds to about 12 keV plasma temperature.

b) From our calculated spin-polarization observables we find that the fusion rate enhancement can be determined experimentally from rather simple measurements in the presently unexplored energy region below E_d = 107 keV. For this purpose, the tensor analyzing power A_{yy} and the polarization transfer coefficient $K_y^{y'}(\vec{d},\vec{n})$ can be used. They are

$$\begin{aligned} A_{yy} &= \frac{1}{2}\,(1 - 2r\cos\delta)\left(1 + \frac{r^2}{2}\right)^{-1} , \\ K_y^{y'} &= -\frac{1}{3}\,(2 + r\cos\delta - r^2)\left(1 + \frac{r^2}{2}\right)^{-1} , \end{aligned} \tag{7}$$

Then, forming the experimental quantity

$$X \equiv (2A_{yy} - 6K_y^{y'}) = (5 - 2r^2)\left(1 + \frac{r^2}{2}\right)^{-1} \quad , \quad r^2 = (5 - X)\left(2 + \frac{X}{2}\right)^{-1} \quad . \tag{8}$$

Since both A_{yy} and $K_y^{y'}$ are isotropic, A_{yy} can be measured at any convenient angle, whereas $K_y^{y'}(\vec{d},\vec{n})$ can be measured most easily at θ = 0 degrees. From the experimentally determined X(E), the enhancement factor f(E) can then be determined via eqs.(5) and (8).

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