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4.4 Coupled Channels Description of Inelastic Scattering with a Dirac Equation

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To describe nucleon scattering, Dirac equation in a coulomb field $V_c(\vec{r})$

$$\left[\frac{\hbar}{i}\vec{\alpha}.\vec{\nabla} + \beta m + V_c(\vec{r}) + \frac{\mu\hbar i}{2m}\beta\vec{\alpha}\left\{\vec{\nabla}\,V_c(\vec{r})\right\}\right]\Psi(r) = E\Psi(\vec{r}) \qquad (1)$$

is generalized into

$$\left[\frac{\hbar}{i}\vec{\alpha}.\vec{\nabla} + \beta\left\{m + V_s(\vec{r})\right\} + V_v(\vec{r}) + \frac{\hbar i}{2m}\beta\vec{\alpha}\left\{\vec{\nabla}.V_T(\vec{r})\right\}\right]\Psi(\vec{r}) = E\Psi(\vec{r}) \qquad (2)$$

where $V_s(\vec{r})$, $V_v(\vec{r})$ and $V_T(\vec{r})$ are three complex potentials of which the real and the imaginary parts are approximated by a Woods-Saxon potential. In eq.(2), $V_v(\vec{r})$ and $V_T(\vec{r})$ include the coulomb potential, multiplied by μ in $V_T(\vec{r})$: μ is the anomalous magnetic moment of the nucleon and m its reduced mass. These three potentials describe the target and its excited states (by inclusion of phonon creation and annihilation operators in the vibrational model or an anisotropic radial dependence in the rotational model). The wave function can be written as

$$\Psi(\vec{r}) = \frac{1}{r}\sum_{\ell jJM}\left[F_{\ell jJ}(r), -i(\vec{\sigma}.\hat{r})G_{\ell jJ}(r)\right]\left[\left\{Y^m_\ell(\hat{r})\,|\sigma>\right\}_j \Psi_T\right]_{JM} \qquad (3)$$

where Ψ_T describes the target, $F_{\ell jJ}(r)$ and $G_{\ell jJ}(r)$ are the large and the small components respectively. Dirac equation becomes :

$$\begin{aligned}
&\hbar\left[\frac{d}{dr} - \frac{K_i}{r}\right]G_i + \left[E-m-V^o_s-V^o_v\right]F_i - \frac{\hbar}{2m}\frac{d\,V^o_T}{dr}G_i = S_i(r)\\
&\hbar\left[\frac{d}{dr} + \frac{K_i}{r}\right]F_i - \left[E+m+V^o_s-V^o_v\right]G_i + \frac{\hbar}{2m}\frac{d\,V^o_T}{dr}F_i = T_i(r)
\end{aligned} \qquad (4)$$

$$\begin{aligned}
S_i(r) &= \sum_{\lambda,j}\mathcal{G}^\lambda_{ij}\left\{\left[V^\lambda_s+V^\lambda_v\right]F_j + \frac{\hbar}{2m}\left[\frac{d}{dr}V^\lambda_T + \frac{K_i-K_j}{r}V^\lambda_T\right]G_j\right\}\\
T_i(r) &= \sum_{\lambda,j}\mathcal{G}^\lambda_{ij}\left\{\left[V^\lambda_s+V^\lambda_v\right]G_j + \frac{\hbar}{2m}\left[\frac{d}{dr}V^\lambda_T + \frac{K_j-K_i}{r}V^\lambda_T\right]F_j\right\}
\end{aligned} \qquad (5)$$

where V^o_s, V^o_v, V^o_T are the monopole terms of V_s, V_v, V_T, $\mathcal{G}^\lambda_{ij}$ the geometrical coefficients which appear for a spin-independent transition in the scattering of spin 1/2 particles and K_i the eigenvalue of $1 + \vec{L}\vec{\sigma}$. An equivalent Schrödinger equation can be written as in ref.[1]. The form is identical except that in eqs.(3) and (4) of ref.[1], $D(\vec{r})$ is replaced by $D(\vec{r})\exp\{V_T(\vec{r})/m\}$. Equations(4-5)can be solved by iteration using the integral version of the ECIS method[2] ; eqs(4) with $S_i = 0$ and $T_i = 0$ are solved to get a regular solution $F^r_i(r)$, $G^r_i(r)$ and a purely outgoing one $F^i_i(r)$, $G^i_i(r)$. For that purpose, the equivalent Schrödinger equation is used (reorientation terms are shifted in the second member in order to obtain potentials of the Schrödinger equation independent of angular momenta). The F's are obtained by dividing[1] by $D(r)^{1/2}$ and the G's by the second eq.(4). With the normalisation of the wronskian $F^r_i(r)\,G^i_i(r) - F^i_i(r)\,G^r_i(r) = \hbar^{-1}$, the iteration procedure is

$$\begin{aligned}
F^{(n)}_i(r) &= \delta_{io}\,F^r_i(r) + F^i_i(r)\int_o^r\left[F^r_i(r')S^{(n)}_i(r') - G^r_i(r')T^{(n)}_i(r')\right]dr'\\
&\quad + F^r_i(r)\int_r^\infty\left[F^i_i(r')S^{(n)}_i(r') - G^i_i(r')T^{(n)}_i(r')\right]dr'
\end{aligned}$$

$$G_i^{(n)} = \delta_{io}\, G_i^r(r) + G_i^i(r) \int_o^r \left[F_i^r(r')S_i^{(n)}(r') - G_i^r(r')T_i^{(n)}(r')\right]dr'$$

$$+ G_i^r(r) \int_r^\infty \left[F_i^i(r')S_i^{(n)}(r') - G_i^i(r')T_i^{(n)}(r')\right]dr' \qquad (6)$$

A precise evaluation of these integrals needs more complex correction terms than for the Schrödinger equations. Computations have been done without the term V_T and are presented here[1].

We use non relativistic coulomb functions. Changing the matching point leaves the inelastic scattering invariant. For the elastic scattering, difference of the long range behaviour between Eq.(2) and the non relativistic coulomb equation can be taken into account by mehtods already described[2]. This difference includes a term $\frac{1}{2m} V_c^2$ and a term $\frac{\ell(\ell+1)}{r^2}\left[\frac{1}{E+m} + \frac{\mu}{m}\right]V_c$ of which the effect is quite the same for Z = 20 when $\mu = 0$ which corresponds to the use of relativistic coulomb functions. Therefore, the use of $\mu = 0$ introduces more error in the result than the use of non relativistic coulomb functions. Relativistic coulomb excitation needs a generalization of the method[2] used in the non-relativistic case because the product of the wave number by the coulomb parameter is no longer a constant.

References

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