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5.7 Restoration of Spin and Isospin Symmetry in the Skyrme Model

John A. Parmentola

West Virginia University
Department of Physics, Hodges Hall
Morgantown, WV 26506

and.

Continuous Electron Beam Accelerator Facility
12070 Jefferson Avenue
Newport News, VA 23606

Through the analysis of large-N QCD (assuming confinement) several unexpected quantitative and qualitative insights have been obtained which are likely to be relevant to the long distance behavior of QCD for N=3. t'Hooft and Witten[1] have shown that QCD is equivalent for any N to a meson theory in which 1/N is the meson coupling constant. The equivalent meson theory is extremely complex, involving an infinite number of elementary meson fields. However, for large N the meson coupling is weak so that some general predictions of the theory can be made, such as Zweig's rule and the existence of a large number (in fact infinite in the large N limit) of narrow meson resonances. Both of these predictions are approximately observed in nature.

For the baryons, Witten has argued that to leading order in the 1/N expansion of QCD baryon masses are of O(N) for large N, baryon sizes are of O(1) and baryon-baryon and meson-baryon scattering amplitudes are of O(1). This behavior strongly suggests that baryons are solitons in large-N QCD and the 1/N expansion of QCD is equal to the semiclassical soliton expansion in the coupling constant, 1/N, of the equivalent meson theory. The quantitative verification of this picture for N=3 has not been achieved; however, the large N limit is expected to be smooth so that it is likely that at least on a qualitative level baryons are solitons in N=3 QCD.

An unexpected result of the 1/N expansion is that baryon properties can be largely understood in terms of meson degrees of freedom and meson phenomenology through solitons. This idea is not totally new (except for the identification of baryons as solitons), since predictions of chiral bag models for strong coupling[2] (bag radii of the order of the proton compton wavelength for which the quark degrees of freedom are suppressed) have also indicated the possibility that baryon physics can be largely understood in terms of meson physics.

Since the equivalent meson theory is very complex, a reasonable approach to testing the idea that baryons are solitons in a meson theory is to choose a meson model which has this feature and incorporates the relevant symmetries of QCD. The Skyrme model is such a model.

According to this approach, the Skyrme model is expected to provide reasonable estimates for the observed properties of the low-lying non-strange baryons. Discrepancies are attributed to the neglect of mesons other than the pion, say the ρ, ω, --- etc., and/or corrections within the semiclassical approximation. So far, some calculations have indicated that the model breaks down for excitations which are more than two pion masses above the nucleon; however, other calculations indicate a wider range of validity (see discussion below).

On the optimistic side, the Skyrme model is expected to describe the same physics as the non-relativistic quark model (for up and down quarks). If the Skyrme model phenomenology is good, i.e. it contains the correct meson physics, then it is reasonable to expect that the Skyrme model Hamiltonian in the semiclassical approximation should have a structural resemblance, i.e. similar terms such as radial kinetic energies, centrifugal energies, radial ascillator potentials, etc., to the non-relativistic quark model. The purpose of this contribution[3] is to report that this may be true provided care is taken in restoring symmetries, both spin and isospin, which are lost at the classical level. Roughly speaking, the usual quark coordinates and momenta are reinterpreted as components of meson fields and their corresponding momenta.

It is well known that the classically determined order parameter or soliton of the Skyrme model breaks spin and isospin symmetry separately even though both symmetries are respected by the model. However, this work[3] is carried out in the semi-classical approximation for which the low-lying non-strange baryons appear as collective excitations that result from the quantization of carefully chosen collective coordinates which restore both spin and isospin symmetry separately. In addition to a very rich spectrum of spin and isospin rotational excitations, there are vibrational excitations which correspond to the compression and expansion of the order parameter or soliton. The lowest lying vibrational excitation is predicted to be 616 Mev above the nucleon which is to be compared to 500 Mev for the N* (Roper).[3]

The starting point in this approach is the Skyrme model Lagrangian which is given by

$$L = \frac{F_\Pi^2}{16} \mathrm{Tr}\{\partial_\mu U \partial^\mu U^\dagger\} + \frac{1}{32e^2} \mathrm{Tr}\{[(\partial_\mu U)U^\dagger, (\partial_\nu U)U^\dagger]^2\} \tag{1}$$

and an anzatz which respects spin and isospin symmetry given by

$$U_c = e^{iF(r) \sum_{asi} \tau_a B^T_{as} Q_s A_{si} \hat{r}_i / Q_o} \tag{2}$$

where F(r) is the classical static solution to the Euler-Lagrange equations, B^T is a 3x3 orthogonal matrix which describes isospin rotations, A is a similar matrix which describes spin rotations, the three Q_r describe vibrations, $\hat{r}_i$ is a spatial unit vector which indicates the selection of p-wave components of the pion field and Q_o is a normalization constant.

Substituting (2) into (1) and keeping terms up to second order in $(Q_r - Q_o)$ the following Hamiltonian is obtained

$$H_c = \frac{2Q_o^2}{\lambda}(K + H_{cb}) + \frac{1}{2}\sum_{ij}(Q_i - Q_o)\omega_{ij}(Q_j - Q_o) + M \tag{3}$$

where Q_o, λ, and the ω_{ij} are easily calculable quantities, M is the classical Skyrmion field energy, the kinetic energy, K, of the vibrational degrees of freedom is given by

$$K = -\sum_r \frac{1}{S} \frac{\partial}{\partial Q_r} S \frac{\partial}{\partial Q_r} \tag{4}$$

with $S = (Q_1^2 - Q_2^2)(Q_2^2 - Q_3^2)(Q_1^2 - Q_3^2)$ the vibrational measure and H_{cb} is a centrifugal barrier in spin and isospin space given by

$$H_{cb} = \frac{1}{4}\{\sum_{rs} \frac{(\hat{J}_{rs} + \hat{I}_{rs})^2}{(Q_r - Q_s)^2} + \frac{(\hat{J}_{rs} - \hat{I}_{rs})^2}{(Q_r + Q_s)^2}\} \tag{5}$$

with $\hat{J}_{rs}$ and $\hat{I}_{rs}$ the cartesian components of body fixed angular momentum and isospin operators respectively.

The Schrödinger equation for H_c can be approximately solved[2] and if F_Π = 129 Mev and e = 5.45 the delta-nucleon mass difference is given by

$$M_\Delta - M_N \simeq 295 \text{ Mev}$$

and the N* (Roper)-nucleon mass difference is given by

$$M_{N*} - M_N \simeq 616 \text{ Mev.}$$

References

1) E. Witten, Nucl. Phys. B223, 422 (1983); ibid, 433.
2) J. A. Parmentola, Phys. Rev. D29, 2563 (1984).
3) J. A. Parmentola, Submitted to Annals of Physics.