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Spin-Polarization Observables in Elastic Electron Scattering from Spin-1/2 Nuclei

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Electron scattering has been used for a long time to probe the electomagnetic structure of nucleons and nuclei. Only recently, however, has serious consideration[1,2] been given to the use of of polarized electrons in experiments other than those designed to search for parity nonconserving effects. This can, perhaps, be understood when it is noted that for scattering at high energies ($\gamma=E/m_e >>1$) the only important spin-dependent terms are those depending on the electron's longitudinal polarization (helicity).[3] Since the scattering cross-section for electrons with longitudinal polarization p_z is given by

$$\sigma=\sigma_0(1+p_zA_z),$$

and since the analyzing power $A_z = 0$ from parity conservation, nothing more is learned from this (one-spin) experiment than is available in the scattering of unpolarized electrons.

However, as has become obvious during the past two decades of hadronic scattering, there are other (two-spin) polarization observables such as polarization-transfer coefficients and spin-correlation coefficients that provide information on the spin-dependence of the interactions which can never be gleaned from differential cross-sections alone. Thus, it is both instructive and useful to view elastic electron scattering from spin-1/2 nuclei within the framework of the general spin structure 1/2 + 1/2 → 1/2 + 1/2.

Since this is the same as that of nucleon-nucleon scattering, for which a complete formalism has been developed, I will use that formalism as the starting point. The M-matrix, defined by $\chi_f = M\chi_i$, where $\chi_i(\chi_f)$ is the initial (final) state two-particle spinor, is given, for non-identical particles, by

$$M = A+B\sigma_{1n}+C\sigma_{2n}+D\sigma_{1n}\sigma_{2n}+E\sigma_{1s}\sigma_{2s}+H\sigma_{1\ell}\sigma_{2\ell} \quad (1)$$

Here $\sigma_{1j} \equiv \vec{\sigma}_1 \cdot \hat{j}$, $\sigma_1(\sigma_2)$ operates on the electron (target) spinor, and (s,n,ℓ) comprise a coordinate system corresponding to (x,y,z) of the Madison convention. Although M is a 4x4 matrix with 16 possible terms, parity conservation and time-reversal symmetry reduce it to the six terms in eq. (1). Even so, with six amplitudes A,B,C,D,E,H there are $6^2 = 36$ independent observables in this nucleon-nucleon scattering. I list the following for later consideration in electron scattering.[4]

$$\begin{aligned}
\sigma &\equiv (00,00)=|A|^2+|C|^2+|H|^2+|B|^2+|D|^2+|E|^2 \\
\sigma A_n^t &\equiv \sigma(0n,00) = 2Re(AC^*+BD^*) \\
\sigma K_{\ell s'} &\equiv \sigma(\ell 0,0s) = 2Im(CH^*+BD^*) \qquad (2)\\
\sigma A_{\ell s} &\equiv \sigma(\ell s,00) = -2Im(CH^*-BD^*) \\
\sigma K_{\ell\ell} &\equiv \sigma(\ell 0,0\ell) = 2Re(AH^*+D^*E) \\
\sigma A_{\ell\ell} &\equiv \sigma(\ell\ell,00) = 2Re(AH^*-D^*E),
\end{aligned}$$

where $(ij,k\ell)$ designates the polarization component of the (initial state electron and target particle, final state electron and recoil particle). The amplitudes in eq. (1) are related to those (a,b,c,d,e,f) of reference 4 by.

$$\begin{aligned}
2A &= a+b \qquad 2C = e-f \qquad 2E = c+d \qquad (3)\\
2B &= e+f \qquad 2D = a-b \qquad 2H = c-d.
\end{aligned}$$

Imposing now on electron scattering the constraint that observables depending on the transverse components of the electron spin tend to zero as $1/\gamma$ (ref. 3), it is found that the amplitudes B=D=E=0.[5] Thus, a remarkable simplification ensues. The number of independent amplitudes is reduced to three, the observables to nine. In this reduction, as is seen from eqs. (2)

$$K_{\ell j} = A_{\ell j}, \quad j = \ell, s \tag{4}$$

From an experimental point of view, this is a very important result since a double-scattering experiment to determine the polarization-transfer coefficient $K_{\ell j}$ can, in principle, always be replaced by a single-scattering experiment to determine the spin-correlation coefficient $A_{\ell j}$. Also, it is noted that with only A,C,H nonzero in eq. (1), the electron helicity-flip terms have vanished so that electron helicity is conserved in this relativistic limit.

Further simplification results from the dynamical nature of the electron scattering process, i.e. one-photon exchange in the plane-wave Born approximation. It is possible to associate the surviving amplitudes with components of the hadronic electromagnetic current. For the very interesting case of electron-nucleon scattering[6] the result is [5]

$$A = H = ikG_M \quad , \quad C = 2MG_E, \tag{5}$$

where $G_E(q^2)$ and $G_M(q^2)$ are the longitudinal (charge) and transverse (magnetic) form factors and M the mass of the nucleon. Thus, the amplitudes are reduced in number to two, with the additional restriction that A is purely imaginary, C real. From eqs. (2) and (5)

$$\begin{aligned} \sigma &= 2|A|^2+|C|^2 \sim 2k^2G_M^2 + 4M^2G_E^2 \\ \sigma A_n^T &= 2\mathrm{Re}A^*C=0 \\ \sigma A_{\ell s} &= 2\mathrm{Im}CA^* \sim -4kMG_MG_E \\ \sigma A_{\ell\ell} &= 2|A|^2 \sim 2k^2G_M^2 \end{aligned} \tag{6}$$

In eqs. (6) the right-hand sides indicate only the form is which the combinations of G_E and G_M appear, omitting the factors which come from the leptonic current. In view of the recognized importance of providing a more accurate determination of the charge form factor of the neutron G_E^n, eqs. (6) show that $A_{\ell s}$ is the observable most sensitive to G_E^n, depending linearly on it.[1,2]

One final consequence of the simplification exhibited in eqs. (6) is that only two of the three (non zero) observables are independent, i.e.

$$\sigma A_{\ell s} = \pm \sqrt{2}\,\sigma[A_{\ell\ell}(1-A_{\ell\ell})]^{1/2}.$$

In summary, it is seen that high-energy electron-nucleon scattering, for example, exhibits a remarkable simplicity when compared with that of nucleon-nucleon scattering. The 6 amplitudes and 36 nucleon-nucleon observables are reduced to 3 and 9, respectively, by the relativistic nature of the electron. The one-photon exchange mechanism finally reduces the number of amplitudes and independent observables to 2. Finally, this treatment serves to extend the spin-polarization "language" of hadronic scattering to that of electron scattering, also.

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