Proc. Int. Symposium on Neutron Optics and Related Research Facilities, Kumatori, 1996
J. Phys. Soc. Jpn. **65** (1996) Suppl. A pp. 25-28

# Synthesized Larmor Precessions and Particle Density Waves

Ferenc MEZEI

*Hahn-Meitner-Institut, BENSC, Glienicker Str. 100, D-14109 Berlin, Germany*

(Received 9 May 1996; accepted 8 July 1996)

Larmor precessions are conventionally looked upon as the effect of a local magnetic field on a localized particle. In contrast, in quantum mechanical particle beams the precessions are described as the interference beating between spin "up" and "down" extended partial waves. Using the combined action of a neutron interferometer and a spin flipper these partial wave states can be prepared in a radically different way from the usual 90° spin flip process. It is found, nevertheless, that the Larmor frequency of these synthesized precessions is equal to the classical value, if observed by an observer traveling along the particle trajectory with the classical velocity of the particle. The same tools are also shown to allow us to prepare neutron beams displaying coherent particle density waves on macroscopic space-time scale, a surprising new quantum phenomenon. Experimental verification of these predictions is in progress at the Hahn-Meitner-Institut.

KEYWORDS: neutron spin, Larmor precession, interference, particle density wave

## 1. Introduction

The vast majority of neutron optical phenomena are exactly analogous to classical wave optics and they can be perfectly described by using 19th century theories developed for the study of light propagation, well before even Maxwell's theory of electromagnetism. In doing this one simply has to infer the wavelength from the de Broglie relation and to establish the neutron refractive indices and absorption coefficients for the materials used. Ultimately neither Bragg's theory of diffraction nor the dynamic diffraction theory goes beyond classical wave optics, although here the atomic structure of the media has to be taken into account instead of a homogeneous refractive index.

Similarly, the description of the behavior of neutron spins in the context of neutron optics (i.e. beam propagation in vacuum, magnetic fields, magnetic materials) can in general be described by the classical mechanical model of a top, i.e. by the equation of motion of a classical mechanical angular momentum under the influence of an eventually time dependent classical mechanical torque. This also applies to phenomena with misleading quantum appearance, such as "dressed neutrons". It has to be stressed, that the classical theory exactly reproduces all experimental observations, and as a matter of fact, these results are identical with those of the quantum theory for non-relativistic energies. The only deviation from classical behaviour in neutron optics is the Stern-Gerlach effect, which remains negligible in most situations. The Stern-Gerlach effect becomes important and observable if the classically calculated magnetostatic forces make the neutron trajectories so strongly dependent on the neutron spin direction, that these modifications of the trajectory become comparable to the geometrical precision of the definition of the beam in a given neutron optical experiment. The manifestation of the Stern-Gerlach effect is a quantized distribution of the magnetostatically modified trajectories in contrast to the classical, continuous one. This quantized distribution goes over to a continuous one for infinite spin values, i.e. for a classical angular moment.

It is the aim of the present paper to examine the consequences of a radically new technique developed in Vienna[1] for preparing polarized neutron states, which has no classical equivalent. As it is well known, neutron spin Larmor precessions can be initiated and subsequently readily observed in a polarized neutron beam by using some kind of a 90° spin flipper device. We will consider here the effects of an alternative way[1] of initiating Larmor precessions with the help of a neutron interferometer in addition to a spin flipper. This technique cannot be conceived in terms of a classical equation of motion for the particle spin moment. In quantum mechanics the spin motion does not only depend on the instantaneous torque, but also on the manner in which the beam has been prepared. We find, nevertheless, that fully independently of the preparation of the particle wave states, the Larmor precession frequency corresponds to the classical value only depending on the local magnetic field along the particle trajectory. We will furthermore show that the interference between partial particle waves of identical spin state, but prepared by using different types of spin flipper devices, leads to the surprising new phenomenon of coherent particle density waves in freely propagating neutron beams. Experiments for the verification of these predictions will also be described.

## 2. Fundamentals

Classical Larmor precessions are described by the equation of motion of the angular momentum $\boldsymbol{S}$ of a particle associated with a magnetic momentum $\mu$. The potential energy $V = -\mu \boldsymbol{B}(\boldsymbol{r},t)$ (where $\boldsymbol{r}$ is the particle position at time $t$) allows one to derive both the instantaneous force $\boldsymbol{F} = -\text{grad}\,(\mu \boldsymbol{B})$ and the instantaneous torque $\boldsymbol{T} = [\mu \times \boldsymbol{B}]$. Thus the Larmor equation of motion is

$$\mathrm{d}\boldsymbol{S}/\mathrm{d}t = \gamma_L[\boldsymbol{S}\times\boldsymbol{B}(\boldsymbol{r},t)]$$

where the Larmor constant $\gamma_L$ describes the ratio of $\mu$ and $S$, e.g. for the neutron with $S = \hbar/2$ it is $2\mu/\hbar \cong 2.9$ kHz/Oe. The solution of this equation in a time independent field $\boldsymbol{B}$ is the well known Larmor precession, the frequency of which only depends on the absolute value of $\boldsymbol{B}$ at the position of the object.

Thus in a homogeneous time independent field the Larmor precession angle in a beam of classical particles is given as

$$\varphi = \varphi_0(t) + \omega_L r/v$$

where $\varphi_0(t)$ is an initial value and $v$ the particle velocity.

In quantum mechanics the spin state of $S = 1/2$ particles will be described as the super-position (i.e. interference) between two fundamental states, say ↑ and ↓. Thus in a magnetic field and quantization direction parallel to the z axis, the x component of the spin of the state $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$

$$\langle S_x \rangle = \hbar \mathrm{Re}\,(\alpha^*\beta)$$

will reveal a precession type time dependence which is determined by the space-time dependence of α and β. Let us assume that we have to do with the superposition of two plane waves $\alpha = a \exp(i\boldsymbol{k}_\uparrow \boldsymbol{r} - iE_\uparrow t/\hbar)$ and $\beta = b \exp(i\boldsymbol{k}_\downarrow \boldsymbol{r} - iE_\downarrow t/\hbar)$. Thus

$$\langle S_x \rangle = \hbar \mathrm{Re}\{a^*b \exp[-i(\boldsymbol{k}_\uparrow - \boldsymbol{k}_\downarrow)\boldsymbol{r} + i(E_\uparrow - E_\downarrow)t/\hbar]\} \quad (1)$$

In order to evaluate this equation, we have to take into account the relation between $\boldsymbol{k}$ and $E$: in a rather slowly varying magnetic field $B = |\boldsymbol{B}|$

$$E_\uparrow = \hbar^2 k_\uparrow^2/2m - \mu B, \qquad E_\downarrow = \hbar^2 k_\downarrow^2/2m + \mu B \quad (2)$$

If we further assume that $\boldsymbol{k}_\uparrow \cong \boldsymbol{k}_\downarrow \cong \boldsymbol{k}$, we can write

$$\boldsymbol{k}_\uparrow^2 - \boldsymbol{k}_\downarrow^2 \cong 2(\boldsymbol{k}_\uparrow - \boldsymbol{k}_\downarrow)\boldsymbol{k}\,. \quad (3)$$

Furthermore the classical particle trajectory is given by the relation

$$\boldsymbol{r} = \boldsymbol{v}\, t + \boldsymbol{r}_0 \quad (4)$$

where $\boldsymbol{v} = \hbar\boldsymbol{k}/m$ is the particle velocity and $\boldsymbol{r}_0$ a constant vector. Inserting Eqs. (2) to (4) into Eq. (1) we find that if we move with the neutron along the classical trajectory the $x$ component of the spin will reveal Larmor precessions with the well known classical frequency $\omega_L = 2\mu B/\hbar$:

$$S_x(t) = \hbar \mathrm{Re}[\exp(-i\omega_L t)\, a^*b \exp(i\varphi_0)] \quad (5)$$

where $\varphi_0$ is a constant.

This result is expected by classical analogy, but the above derivation implies a rather surprising fact: By superposing two arbitrary free particle states with opposite spins with the only restriction that the two trajectories should be rather close to each other in both space and time, we will observe the classical Larmor precessions along this trajectory. These precessions in the quantum mechanical case are the result of beating effects between the two particle waves due to the difference in the wave number $\boldsymbol{k}$ and/or in the energy $E$ between the ↑ and ↓ superposed states. In practice, of course, it is quite difficult to produce coherently superposed states. In working with polarized neutron beams this only happens for particle waves with different spin states originating from the same initial wave which was split in the spin variable space by e.g. a 90° dc spin flipper, as in Neutron Spin Echo spectroscopy[2]. When using dc flippers, the energy conservation requirement will ensure that $E_\uparrow = E_\downarrow$ all the time[3] and consequently in view of Eq. (2) there is a difference between $\boldsymbol{k}_\uparrow$ and $\boldsymbol{k}_\downarrow$ if the magnetic field is not zero. For NMR type radio frequency (rf) flippers the energy conservation implies energy transfer between the particle and the rf field. The above algebra shows that the conventional Larmor behaviour does not only apply to coherent superpositions of spin states produced by spin flipping processes, but also to *arbitrary* coherent superpositions which cannot be realized by usual experimental techniques.

For the more general case we also have to consider more complicated wave functions than single plane waves. For the superposition of wave packets with opposite spin directions,

$$\Psi_\uparrow = \sum a_{k\uparrow} \exp(i\boldsymbol{k}_\uparrow \boldsymbol{r} - iE_\uparrow t/\hbar) \quad (6)$$

and $\Psi_\downarrow$ (similar to (6) with the coefficients denoted by $b$ instead of $a$), Eq. (5) will take the more general form:

$$S_x(t) = \hbar \mathrm{Re}\Big[\exp(-i\omega_L t) \sum_{k\uparrow,k\downarrow} \langle a_{k\uparrow}^* b_{k\downarrow} \exp(\varphi_0)\rangle\Big] \quad (7)$$

Each term in the sum represents the average taken over the beam, i.e. over an ensemble of states of type (6). It is a basic, fundamental fact in all particle beam interference experiments that no coherence can be obtained in a beam between different $\boldsymbol{k}$ eigenstate components of the initial particle states. In connection with Eq. (7) this implies, that only those averages under the summation are different from zero in which $\boldsymbol{k}_\uparrow$ and $\boldsymbol{k}_\downarrow$ correspond to states split from a common initial plane wave component of the initial particle state.

## 3. Experimental realization

We will consider here the possible consequences of producing the coherent pairs of states in a superposition of the type of Eq. (6) not by spin flipping only, but by the combined action of real space beam splitting in an interferometer and spin flipping. We have to start with the fact that in the initial particle beam states all $\boldsymbol{k} \neq \boldsymbol{k}'$ components are incoherent. (In the usual normalization scheme with periodic boundary conditions over a large volume, only discrete values of $\boldsymbol{k}$ are admitted. This remains valid in an infinite volume, continuous $\boldsymbol{k}$ space representation if it is considered as the limit for a large volume, so that the meaning of equality in the $\boldsymbol{k}$ space stays unambiguous. Another interpretation of a continuous representation would bring up the problem of "how

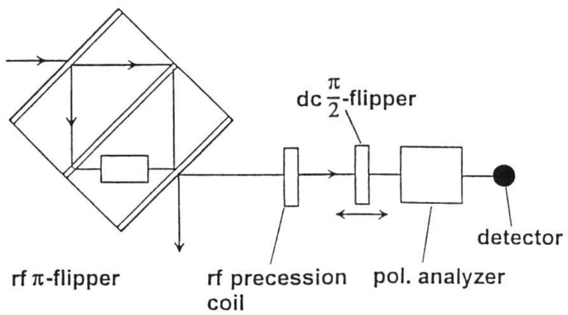


Fig. 1. Experimental set-up for the verification of Larmor precession behaviour using a Bonse-Hart neutron interferometer and spin flipping for the preparation of coherent neutron states with opposite spin directions.

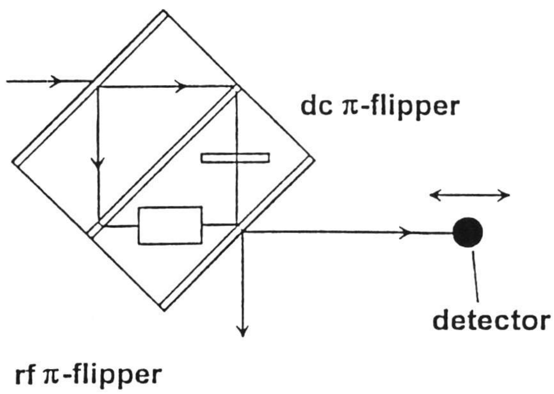


Fig. 2. Experimental set-up for the production of coherent particle density waves in the freely propagating beam after the interferometer, due to the energy difference between the two partial waves flipped inside the interferometer by different type of spin flipper devices.

exactly" $k = k'$ means equality, and this is an unexplored territory.)

Figure 1. shows the scheme of the experimental set-up we wish to consider. It is based on the technique described in Ref. 1. Using an rf π-flipper in order to produce the ↓ spin component of the outgoing beam means that the flip does not change the wavenumber[3]. The ↑ component, separated in the interferometer, does not go through the flipper, it changes neither spin nor wavenumber. In view of Eq. (5) and (7) we do not expect any deviation from the classical Larmor precession behaviour, in spite of the fact that we have to do with a superposition of opposite spin states which cannot be achieved by spin flipping only. This conclusion remains valid even if we take into account that the rf flipper represents a time dependent term in the Hamiltonian. The experimental observation of the precession occurs in the free particle propagation range around the dc π/2 flipper in a static magnetic field, where Eq. (6) and (7) apply. What actually happens in the rf field region is that the interaction between the rf field and the particle eventually results into transitions from an initial plane wave state into other ones with different energies, thus an initial plane wave can emerge as a superposition of several waves. This possibility is included in Eq. (6), where in a static magnetic field the Schrödinger equation requires that the coefficients $a_{k\uparrow}$ are independent of time. Since the energy (frequency) of the rf field is in practical situations of a flipper some 6 orders of magnitude smaller than the neutron kinetic energy, for the precision of the observation of the Larmor precession, as shown in the figure, the neutron energy changes in the rf field are fully negligible and we will have a common velocity $\boldsymbol{v}$, as in the approximation used in connection with Eqs. (3) and (4). In practice the investigation of Larmor precessions "along the classical particle trajectory" implies the observation of the $S_x$ spin component both as a function of time at a given position of the π/2-flipper (e.g. by adding a time dependent rf precession phase shifter coil) and as a function of space at a given time (with respect to the phase of the rf flipper) by displacing the π/2 flipper along the beam direction, cf. Fig 1.

The combination of an interferometer and spin flippers can also be used for the preparation of neutron beams with coherent wave components having different wavenumbers $\boldsymbol{k}$ but the same spin wave function. This has never been experimentally achieved before, nor has a method been proposed how to do it. (In experiments, where such coherence plays no role, it can, of course, always be claimed that coherence exists, but these claims only are unproved assumptions.) In the proposed experimental set-up in Fig. 2. the initially ↑ spin partial wave going through the dc flipper (upper path in the figure ) leaves the interferometer with ↓ spin but with no change in the total energy, while the energy of the partial wave going through the rf flipper (and also flipped into the ↓ state) changes by $2\mu\boldsymbol{B}$, where $\boldsymbol{B}$ is the dc magnetic field inside the rf flipper[3]. This energy difference between the two partial waves is maintained in the free space propagation after the interferometer in the recombined beam.

The particle density in this beam will be given, analogously to Eq. (1) (with the indices ↑ and ↓ replaced by $u$ and $\ell$ for the upper and lower beam paths in the interferometer, respectively, cf. Fig. 2) as:

$$\rho(\boldsymbol{r},t) = a^*a + b^*b + \\ +\mathrm{Re}\{a^*b \exp[-i(\boldsymbol{k}_u - \boldsymbol{k}_\ell)\boldsymbol{r} + i(E_u - E_\ell)t/\hbar]\}. \quad (8)$$

Here, as pointed out above, $E_u - E_\ell = -2\mu\boldsymbol{B}$ (where $\boldsymbol{B}$ is the field at the rf flipper) and thus in view of Eq. (2) and (3)

$$\boldsymbol{v}(\boldsymbol{k}_u - \boldsymbol{k}_\ell) = (E_u - E_\ell)/\hbar \quad (9)$$

with $m\boldsymbol{v}/\hbar \cong \boldsymbol{k}_u \cong \boldsymbol{k}_\ell$. Thus Eq. (8) describes a particle density wave with a phase velocity $\boldsymbol{v}$ and a wavelength which corresponds to the Larmor precession period[2] in a free neutron beam in a static field $\boldsymbol{B}$. Equation (8) can be easily generalized to initial particle states of the type (6) and the result will be analogous to Eq. (7). In practical terms the single $\boldsymbol{k}$ approximation (8) remains valid if the initial beam is sufficiently monochromatic for keeping the density waves for different velocities $\boldsymbol{v}$ in phase. This condition exactly corresponds to the experimentally well established technique of observing Larmor precessions in neutron beams[2] because we have to do with the same oscillation periods, cf. Eq. (9). Actually, since in the approximation used in Eq. (9) the phase velocity of the density waves is identical for ↑ and ↓ spin initial states, this experiment does not require a polarized incoming beam.

**4. Conclusion**

In contrast to the classical description of the spin behaviour in particle beams, in quantum mechanics the spin motion not only depends on the instantaneous local forces and torques, but also on the way the particle states have been prepared. The use of neutron interferometry opens up new experimental opportunities for beam preparation by allowing us to perform different spin manipulations on the two coherent partial waves separated in space inside the interferometer. The two experiments proposed here are in preparation at the Hahn-Meitner-Institut. The one shown in Fig. 1 is aimed at the verification of the expected classical Larmor precession behavior in the free neutron beam after the interferometer, in spite of the fundamentally non-classical beam preparation. The experiment shown in Fig. 2 has the goal of producing – for the first time – coherent superposition of neutron waves with different wavenumbers and a common spin state. This is proposed to be achieved with the help of the different forces acting on the neutron in the different types of spin flipper devices used in the two arms of the interferometer. The result is a surprising new quantum phenomenon: coherent particle density waves on macroscopic space-time scale.

---

1) G.Badurek, H. Rauch and J. Summhammer, *Phys. Rev. Lett.* 51 (1983) 1015

2) F. Mezei, ed. *Neutron Spin Echo* (Spinger Verlag, Heidelberg, 1980); see also the contribution of R. Gähler et. al. in this proceedings.

3) F. Mezei; *Physica* B 151 (1988) 74