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Neutron Interferometry in Non- Inertial Reference Frames

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The phase shift of a neutron in an interferometer containing uniformly accelerating matter is studied theoretically using semiclassical techniques. Both a neutron source at rest and an inertial source instantaneously comoving with a slab of matter are considered, and the results are compared with those obtained assuming a constant gravitational field in the same geometry. It is shown that the interferometric phase difference is dependent on the position of the matter in the interferometer and velocity of the matter relative to the interferometer at the time the neutron is detected. Two consequences of this conclusion are: (1) the phase difference measured by an interferometer containing samples of matter of differing types that are accelerating or in a gravitational field will apparently be affected by the order of the samples due to the position-dependence of the phase shift caused by each sample, and (2) for matter accelerating in an interferometer at rest the phase difference will be time-dependent. Experiments designed to measure these effects are described.

KEYWORDS: Acceleration, Equivalence Principle, Neutron Interferometry, Fizeau Effect

One of the fundamental concepts of physics is the equivalence principle- the idea that the effects of gravity and acceleration on the trajectory of a classical particle are locally indistinguishable. In the realm of classical physics, experiments have verified the validity of this idea to very high precision.[1)] A way to state the analogous idea for quantum mechanics is that the wave function of a quantum mechanical system in a uniform gravitational field g is indistinguishable from that of the same system uniformly accelerated with acceleration $-g$. As the wave function is in general complex, it may be written as the product of two functions, a real probability amplitude and a complex phase. It has been demonstrated that the probability density of neutrons in the Earth's gravitational field follows the same parabolic trajectory as a classical particle of the same inertial mass,[2)] establishing the equivalence of inertial and gravitational mass with regards to quantum mechanical probability amplitudes with an uncertainty of 3 parts in 10^4.

The question of the equivalence of gravity and acceleration on quantum mechanical phase is more subtle, as the phase of a particle has no classical analog. Since detectors in quantum mechanical experiments involving massive particles are particle detectors and thus measure probability densities, the phase is not directly measurable. It is, however, indirectly accessible in the form of interferometric phase differences as have been observed for neutrons,[3)] electrons,[4)] atoms,[5)] and more recently, molecules.[6)] Since the quantum mechanical phase is observable only in the form of phase differences, it is the effects of gravity and acceleration on these phase differences rather than on the absolute phase of the wave function that one would expect to be indistinguishable.

For neutron deBroglie waves, the phase difference due to gravity has been studied by tilting a silicon perfect crystal interferometer at various angles with respect to the Earth's surface in a series of experiments.[7-11)] Similarly, the phase shift due to constant acceleration has been measured by driving a similar neutron interferometer sinusoidally and taking data stroboscopically at the endpoints of the motion for various frequencies.[12)] In both of these experiments the phase of the interferogram was determined by rotating a slab of matter in the two subbeams of the interferometer. Both experiments agreed with the equivalence principle to within a few percent, but later, more precise measurements[10,11)] of the gravitationally induced quantum phase shift have shown some interesting discrepancies at the 1 percent level. Neither experiment was sufficiently precise to measure phase effects due to apparent couplings between gravity (or acceleration) and the optical potential of the material slab or to detect terms beyond the first order in g. It has been recently suggested [13)] that the equivalence principle may not hold in higher order for these effects. It is the calculation and measurement of these effects that is the goal of this work.

In order to compare the effects of acceleration and gravity, we consider the simple theoretical interferometer shown in Fig.1, consisting of a source emitting a particle with initial kinetic energy ε_0 and momentum $p_0 = \hbar k_0$ in the source frame, a slab of material with constant optical potential U_0 and thickness d in path II of the interferometer, and a detector. In order to neglect any time the neutron

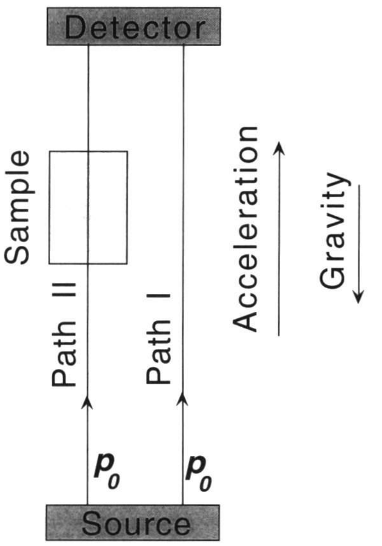

Figure 1. A schematic diagram of the theoretical interferometer geometry used in calculation. This geometry allows curvature effects to be neglected.

spends traveling horizontally, the lateral separation between paths I and II is assumed to be negligible.

In order to calculate the semiclassical phase shift in this interferometer we integrate the Lagrangian of the system along the classical subbeam trajectories in the interferometer[14,15] in the source frame. We first consider the case of a uniformly accelerating slab in conjunction with a comoving accelerating source. Instead of treating the Lagrangian by inferring a potential based on the trajectory for an accelerating source (a method which essentially assumes the equivalence principle) the Lagrangian is modified by the Galilean coordinate transformation $z' = z - \frac{gt^2}{2}$ so that the free particle Lagrangian $L = \frac{m\dot{z}^2}{2}$ becomes $L' = \frac{m(\dot{z}' + gt)^2}{2}$ in the accelerated frame. Although this method yields a different answer for the phase accumulated in each path than the method using the Lagrangian equivalent to that due to constant gravity, the important, measurable quantity, the phase difference is the same and is given by

$$\Delta\Phi = -\frac{2k_0\varepsilon_0}{3mg}\left\{\left(1 - \frac{mg}{\varepsilon_0}(z_0 + d) - \frac{U_0}{\varepsilon_0}\right)^{3/2} - \left(1 - \frac{mg}{\varepsilon_0}(z_0 + d)\right)^{3/2} - \left(1 - \frac{mgz_0}{\varepsilon_0} - \frac{U_0}{\varepsilon_0}\right)^{3/2} + \left(1 - \frac{mgz_0}{\varepsilon_0}\right)^{3/2}\right\}, \quad (1)$$

where z_0 is the position of the slab. The solution of the time-dependent Schrödinger equation (neglecting reflection) reduces to this expression if the neutron is far from the turning point of its classical, parabolic trajectory in the accelerated frame.

The case of an accelerating slab in a beam produced by a source at rest is more complicated, and so is treated only to leading order in the optical potential U_0. If the position of the slab in the source frame as a function of time is $z_0 + \frac{1}{2}gt^2$, the phase difference is

$$\Delta\Phi = \frac{k_0U_0}{mg}\left\{\left(1 - \frac{mg}{\varepsilon_0}\left(z_0 + \frac{\hbar k_0\tau}{m} + d\right)\right)^{1/2} - \left(1 - \frac{mg}{\varepsilon_0}\left(z_0 + \frac{\hbar k_0\tau}{m}\right)\right)^{1/2}\right\}, \quad (2)$$

where τ is the time at which the particle is emitted by the source (related by geometry to its time of detection T). Note that while this expression is valid to the stated order, it ignores essential physics. When the neutron enters the accelerating slab, its frequency is Doppler shifted due to the motion of the slab boundary, and when it exits it is again Doppler shifted as has been discussed previously in the literature in discussions of the neutron Fizeau effect.[16-19] However, these Doppler shifts do not exactly cancel here as they do for the Fizeau effect, since the speed of the slab boundary when the neutron exits the slab is different from the speed when it entered. This difference will manifest itself in higher orders in the optical potential as a loss of interference contrast and a dependence of the phase on the position of the detector. This assumes that the neutron is only affected by the motion of the boundaries of the sample, not by the motion of the nuclei comprising the sample.

The situation of a uniformly accelerating Mach- Zehnder interferometer in conjunction with both a comoving and an inertial source has been discussed previously.[20] Although the geometry considered and the method of calculation used differ, this previous work also shows that acceleration of one component of the system relative to another introduces a time-dependence into the measured interferometric phase.

If the phase differences given by eqs.(1) and (2) are expanded in a Taylor series, permissible by the same small optical potential and acceleration which enable the use of semiclassical methods, both contain terms proportional to U_0mg. These terms we refer to as "coupling terms". If two slabs S1 and S2 of differing material are placed in one path of an interferometer in any of the situations described in this paper, the phase shift measured will vary with the order of the slabs (first S1 then S2, or first S2 then S1) due to these terms. This non-commutative effect is surprising and has not yet been observed experimentally.

The reason for this apparent coupling between the neutron-nuclear optical potential and gravity can easily be seen from the quadratic form of the dispersion relation for the neutron. The spatially-dependent index of refraction of a neutron traversing a slab of matter permeated by the gravitational field is

$$n(z) = \left(1 - \frac{U_0 + mgz}{\varepsilon_0}\right)^{1/2}, \quad (3)$$

or

$$n(z) \cong 1 - \frac{U_0 + mgz}{2\varepsilon_0} - \frac{U_0^2 + 2U_0mgz + (mgz)^2}{8\varepsilon_0^2} \quad (4)$$

to second order in the potential energy. Thus, the phase shift,

$$\Delta\Phi = \int k_0n(z)dz, \quad (5)$$

contains a term proportional to U_0mgz that we refer to as a "coupling term". To date, neutron interferometry experiments have only been sensitive to the linear terms.

An experiment to measure the phase shift due to the apparent coupling of the gravitational and neutron-nuclear optical potentials seems difficult at present. Such an experiment is described in Fig.2. Two identical slabs of matter are placed so that one is in the horizontal portion of

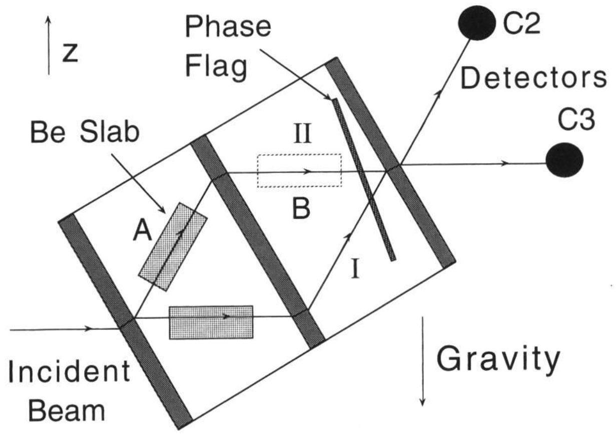

Figure 2. A schematic diagram of the experiment to measure the phase shift of the neutron due to the combined action of matter and gravity.

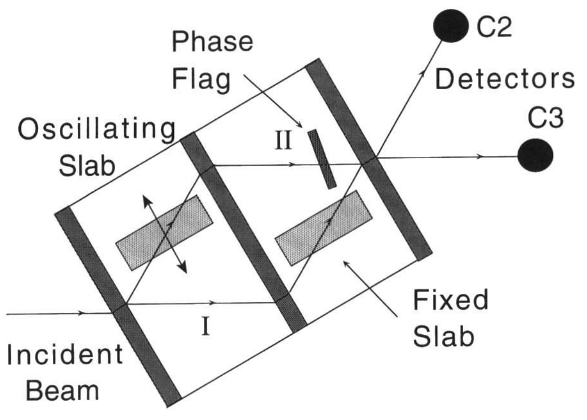

Figure 3. A schematic diagram of the experiment to measure the phase shift of the neutron due to the acceleration of matter.

path I and the other is in first position A and then position B in path II of a tilted neutron interferometer. A thin phase flag is rotated to generate interferograms and the phases of the interferograms observed for the slab at positions A and B are compared. The slab in path I is again used to prevent loss of contrast.[21] Even considering the optimal situation of using our largest interferometer and slabs made of crystalline beryllium (the element with the largest thermal neutron scattering length density) 2.5 cm thick (half of the interferometer blade separation), the expected phase difference as determined using the WKB approximation is only 0.2 mrad. Due to phase stability considerations, our current experimental resolution is limited to approximately 1 mrad. Further refinements to the experimental setup, such as manufacture of a much larger interferometer, are necessary before this experiment can be attempted.

We are at present assembling the apparatus at MURR for an experiment to test the validity of eq. (2). The experiment will be configured as illustrated in Fig.3. Identical sapphire crystals 1 cm thick are placed in each path in a LLL perfect silicon crystal Mach-Zehnder neutron interferometer. The crystal in path II is driven sinusoidally along a line with an amplitude of motion of about 1 cm and a frequency ranging from 0 to 30 Hz by means of a stepper motor attached to a pair of crossed linear bearings while the crystal in path I remains fixed. The data are then collected by time-of-flight methods synchronized with the motion of the first crystal, and time-dependent interferograms will be measured. A thin aluminum phase flag will be set to maximize the amplitude of the time-dependent phase oscillations. The expression (2) for an inertial source and an accelerating sample of matter will be tested against the data at times corresponding to the turning points of the motion.

Since there are identical slabs in both paths, there will be no loss of contrast due to longitudinal coherence length effects.[21] The slabs should have no effect on the phase difference except those introduced by the motion of the slab in path II. To the anticipated resolution of this experiment this phase difference is proportional to the difference in time which the neutron would spend in each slab or, equivalently, the different effective thicknesses of the slabs.

The time dependent phase shift expected can be calculated for the whole period of oscillation by the same techniques. If the position of the slab in path II as a function of time is $z_0+A\sin\omega t$, then the resulting interferometric phase difference is

$$\Delta\Phi(T) = \frac{Ad\omega U_0 \sin \omega t_i(T)}{\hbar\varepsilon_0 \sin^2 \theta_B}, \tag{6}$$

where A is the amplitude of motion, ω angular frequency of oscillation, and θ_B the Bragg angle of the interferometer. The time $t_i(T)$ is the time at which the neutron enters the slab in path II, given by

$$t_i(T) = \frac{m}{\hbar k_0 \sin\theta_B}\left[z_0 + A\sin\omega t_i(T)\right] + T - \frac{mL}{\hbar k_0}, \tag{7}$$

where T is the time at which the neutron is detected and L is the path length along path II from the first blade of the interferometer to the detector. Here τ in eq. (2) corresponds to the time when the neutron enters the interferometer at point A. This leads to an interferogram of the form

$$I(T) = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{Ad\omega U_0 \sin\omega t_i(T)}{\hbar\varepsilon_0 \sin^2\theta_B} + \phi_0\right), \tag{8}$$

where ϕ_0 is the offset phase set using the aluminum phase flag. This predicted interferogram is illustrated in Fig.4 for a number of different frequencies.

Since the phase of the interferogram is a periodic function with the same period as the slab motion, it can be Fourier decomposed into harmonics of the slab oscillation frequency. In this way the predicted interferogram for this experiment can be rewritten

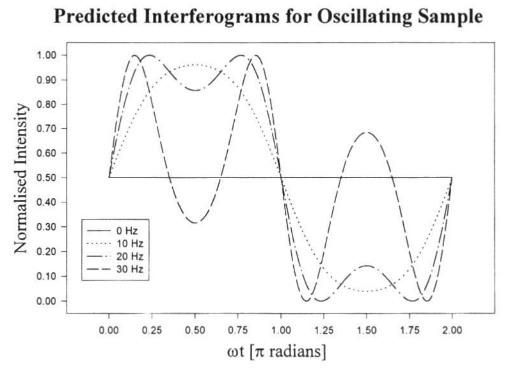

Figure 4. The time-dependent interferograms for the experimetal arrangement of Fig.3 predicted by eq.(8). The offset phase is set to $-\pi/2$ to maximize visibility at low frequency.

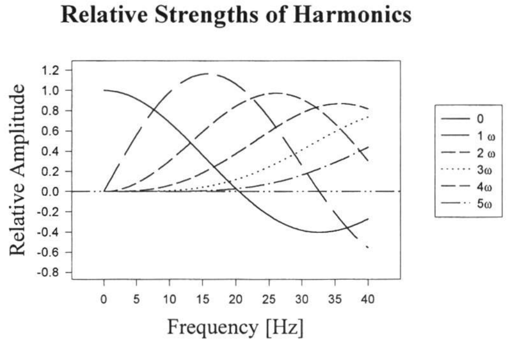

Figure 5. The relative strengths of the harmonics of ω in eq. (9). The offset phase is set to $-\pi/4$ to equalize the strengths of the even and odd harmonics and the amplitudes are nomalised.

$$I(T) = \frac{1}{2} - \sin\phi_0 \sum_{n=1}^{\infty} J_{2n+1}(b\omega)\sin(2n+1)\omega t_i(T)$$
$$+\cos\phi_0\left(\frac{1}{2}J_0(b\omega) + \sum_{n=1}^{\infty} J_{2n}(b\omega)\cos 2n\omega t_i(T)\right), \tag{9}$$

where

$$b = \frac{AdU_0}{\hbar\varepsilon_0 \sin^2\theta_B}. \tag{10}$$

For the described experiment, $b = 0.1173$ s. The relative strengths of the various harmonics as a function of frequency is shown in Fig.5. Since the strengths of a harmonic component is related to the transition probabilities of the neutron to states where its frequency has been modified by the addition or subtraction of that harmonic of the oscillation frequency, the experiment also has an interpretation in terms of quasi-stationary states [22] and phonon exchange similar to that of the recent multiphoton exchange experiment.[23,24]

The nonlinear form of the index of refraction for neutrons leads to an apparent coupling between the neutron-nuclear optical potential and gravity or acceleration in the phase differences measured with a neutron interferometer. Although small, these effects should be visible for a neutron interacting with an accelerating slab as described in this paper. The equivalence principle is satisfied theoretically to the accuracy of the semiclassical techniques used. If the source and the slab are not comoving the interferometric phase difference is found to be time-dependent, an interesting kinematical effect unrelated to the equivalence principle.

Periodic motion of a slab in one path of an interferometer produces a time-dependent interferogram that can be decomposed into harmonics of the slab motion, suggesting that the frequency of the incident beam is chirped by harmonics of the frequency of the slab motion due to interaction with the slab.

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