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Gravity and Inertia in Neutron Crystal Optics and VCN Interferometry

*,**K. Raum, *,†M. Weber, **R. Gähler, *A. Zeilinger

**Institut für Experimental Physik, Universität Innsbruck, Austria.*
***Fakultät für Physik E21, Technische Universität München, Germany*
†Institut Laue Langevin, Grenoble, France

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We present results of experiments where the deflection of neutrons in a perfect crystal due to gravity and due to a Coriolis force in a rotating crystal was observed. They are based on the fact that a neutron inside a perfect crystal shows an effective mass more than five orders of magnitude smaller than its rest mass. These experiments allow a test of the equivalence principle in the quantum mechanical limit. The precise measurement of the phase shift due to gravity with the very cold neutron interferometer for a neutron wavelength of 10nm, formed by three transmission phase gratings with a distance of up to 2 m are also discussed.

KEYWORDS: gravity, inertia, VCN interferometry, neutron crystal optics

§.1 Introduction

There are only few experiments which for their interpretation involve both quantum mechanics and the equivalence principle of relativity theory. This holds for the so called COW experiments and for our experiments: the deflection of neutrons in a crystal due to gravity, or by a Coriolis force. In the COW experiments the phase shift due to the gravitational field of the earth was measured with a perfecct crystal neutron interferometer.[1] The latest revision of COW's data by Werner et.al. (1988) shows a deviation from theory of 0.8%. This is significantly larger than the measurement accuracy of 0.1% and there is a strong need for experimental clarification.[2,3] Therefore we will try to measure his phase shift with an interferometer for very cold neutrons avoiding the major experimental and theoretical difficulties of the former experiments.

Another approach to observe gravitational or inertial effects in the quantum limit is the effective mass enhanced deflection of neutrons due to gravity or by a Coriolis force in a perfect crystal.[4] This deflection is enhanced by more than 5 orders of magnitude over the expected deflection in free space. A remarkable feature of the experiments is the existence of two effective mass states inside the crystal, one with a positive and one with a negative effective mass. Neutrons in the negative effective mass state are accelerated opposite to a force.

The effective mass m^* can be calculated from the curvature of the dispersion relation $\omega(\boldsymbol{K})$, where $\boldsymbol{K}$ is the wave vector inside the crystal.[5] This relation can be derived by solving the Schrödinger equation within the periodic crystal potential. Assuming neutrons traveling close to the Bragg angle Θ_B for one single set of parallel lattice planes characterized by the reciprocal lattice vector $\boldsymbol{G}$, it is sufficient to use an ansatz for the wave function which is a superposition of two plane waves, one propagating approximately parallel to the primary neutron beam outside the crystal, and one in the Bragg reflected direction. A detailed calculation shows, that two states of that form exist for one set of boundary conditions. From this calculation the acceleration a^* in the crystal parallel to $\boldsymbol{G}$ due to an acceleration $\boldsymbol{a}$ of the free neutron in the rest frame of the crystal is given by

$$a^* \equiv \pm(E_G / (2V_G))(\boldsymbol{a} \cdot \hat{G}) \qquad (1)$$

Here V_G is the Fourier coefficient of the crystal potential corresponding to $\boldsymbol{G}$, $\hat{G}$ the unit vector in the direction of $\boldsymbol{G}$ and E_G the kinetic energy of a neutron with a wavelength equal to the lattice spacing $2\pi/G$. The two possible signs of the acceleration a^* correspond to the two effective mass states mentioned above. For the Si(220) reflex, which was used in both experiments, the acceleration a^* up to 2.1×10^5 times the acceleration a of the free neutron. Eq.1 is valid for small deviations from the Bragg angle. If this deviation gets larger the acceleration decreases.[4,5] Due to the high sensitivity of the direction of neutron propagation to a small change in the direction of incidence, the neutron beam has to be highly collimated. We use a setup consisting of two silicon single crystals (L = 52mm) on a common base cut from one crystal to provide parallel lattice planes in both parts as shown in Fig.1. The first crystal together with two Cd slits at the entrance and the exit acts as a collimator. As the two effective mass states are deflected in opposite directions, the trajectories for the two states are separated in the second crystal. The spatial separation d of the two effective mass states measured perpendicular to the beams due to an acceleration a^* is given by

$$d = 2a^*(L/v_d)^2\cos(\Theta_B)(1+s/L) \ . \qquad (2)$$

Here L is the length of the two crystals, s the width of the gap between them (see Fig.1) and $v_d = v\cos(\Theta_B)$ the drift velocity of the neutrons parallel to the lattice planes. The factor ($1 + s/L$) is due to the deflection in the gap between the two crystals because the deflection here changes the deviation from the Bragg angle which is related to the initial slope of the neutron trajectories[4] in the second crystal. The formula is valid for a force perpendicular to the neutron trajectory in the gap.

§.2 Deflection due to gravity

The gravity experiment was carried out at the T13A test facility of the ILL. The first slit was illuminated by a monochromatic neutron beam with a mean wavelength of 2.35A provided by a graphite monochromator. The silicon

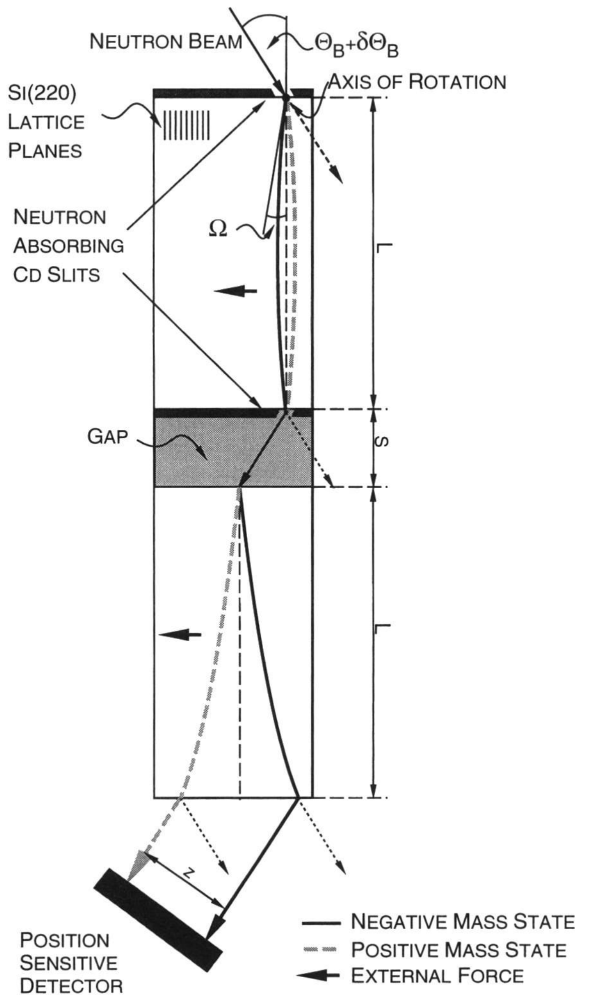

Fig. 1. Neutron trajectories in the gravity experiment.. The fine dashed lines are the neutron trajectories without external force; the fat line trajectories are valid with external force. The dimensions were L = 52.3mm and s = 9.6mm. The arrangement of the slits and the neutron trajectories in the Coriolis experiment can be found in Ref.4. The slit width was 1.6 mm in the Coriolis experiment and 1.5 mm in the gravity experiment.

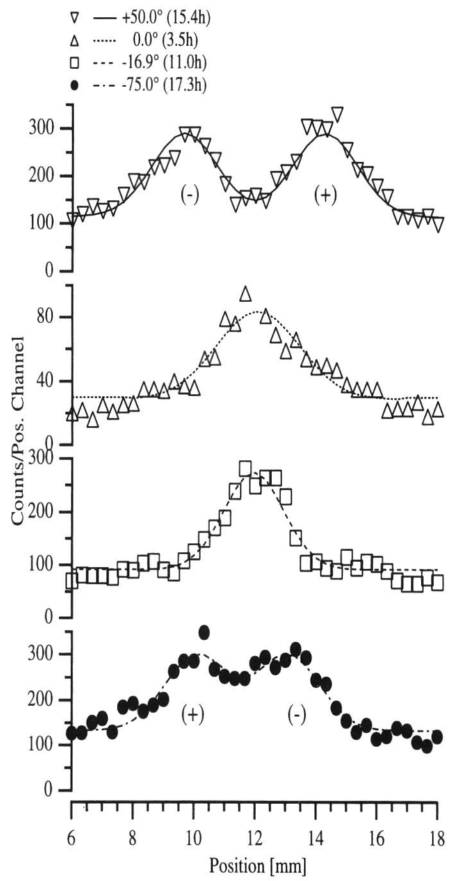

Fig. 2. Counts as a function of position on the detector at -75°, -16.9°, and 50° tilting angle. The + sign indicates the positive effective mass state. The - sign the negative effective mass state.

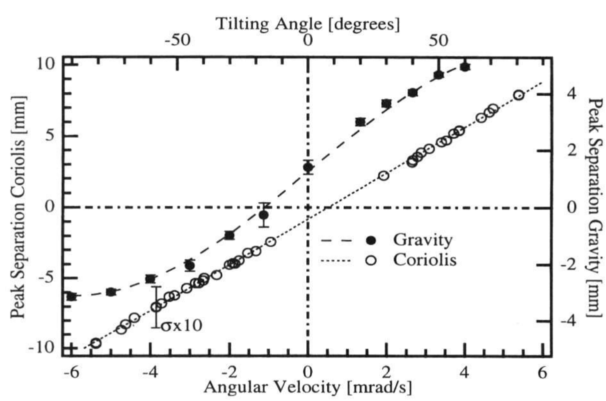

Fig. 3. Fig. 2: Peak separation as a function of the tilting angle (Gravity) and angular velocity (Coriolis).

crystals and the detector could be tilted around the axis of the incoming neutron beam. As only the component of Earth's acceleration $\boldsymbol{g}$ parallel to $\boldsymbol{G}$ leads to a detectable deflection, the separation of the two effective mass peaks is proportional to the sine of the tilting angle ϕ. After passing through the crystal the neutrons were detected in a position sensitive detector. Due to the high sensitivity of the neutron trajectories to the deviation from the Bragg angle, the experiment is very sensitive to a bending of the crystal under its own weight. Bending of the interferometer crystals was one of the major experimental uncertainties of the COW experiments. In our experiment it was avoided by placing the crystal in a fluid of equal density as silicon ($ZnBr_2$ in D_2O). The predicted separation of the two effective mass states of $d/\sin\phi = (4.737 \pm 0.011)$mm agreed with the measured value of $(d / \sin\phi)_{exp} = (4.740 \pm 0.05)$ mm. The error in the theoretical value arises from uncertainties in the wavelength. The experimental errors are mainly due to counting statistics. In passing by we note that the separation of the two peaks depends on the ratio of the inertial mass m_i and the heavy mass m_g of the neutron. The experimental results (see Fig.3) showed a constant offset of 1.3 mm of the separation of the two effective mass peaks. It can be explained by a small intrinsic bending of the crystal. An offset of the same order of magnitude but opposite direction was observed in all former experiments with the same crystal but with a different arrangement of the collimating slits where the Bragg diffracted beam was used after the first crystal. The slit positions were changed because Earth's acceleration is perpendicular to the forward diffracted beam in the gap for all tilting angles.

§.3 Deflection due to a Coriolis force

In the Coriolis experiment which was carried out at the NIST research reactor, the crystal was rotated around the vertical axis of the first collimating slit. The rotation of the crystal at an angular velocity Ω results in a Coriolis acceleration parallel to G of $a^* = 2\ (m/m^*)\ \Omega v_d$.

As the crystal had to be kept under the Bragg-angle with respect to the incoming neutron beam, a rotatory oscillation with an amplitude much smaller than the divergence of the primary beam was used. This oscillation was driven by a Mößbauer velocity transducer at the rear end of the crystal. Below the first collimating slit the crystal support was connected to the fixed parts of the setup by a flexure, thus the linear motion of the transducer resulted in a rotation of the crystal. The amplitude of the oscillation was adjusted between 3.6 mrad/s and 6 mrad/s and the frequency between 13Hz and 26Hz.

A calculation of the peak separation as a function of the angular velocity of the rotation results in a linear dependence with a slope $d / \Omega = (\ 1.622 \pm 0.008\ )$ m/ (rad/s) (see Eq.2, λ = 2.35Å). This has to be compared to the measured slope of $(d\ /\ \Omega)_{exp}$ = (1.609±0.014) m/(rad/s). The maximum acceleration acting on the crystal was less than 0.1m/s^2 compared to 20 m/s^2 on the neutrons. To check whether the crystal was bent by this acceleration we made measurements at different oscillation frequencies. As the acceleration of the crystal depends quadratically on the frequency and the acceleration of the neutrons linearly, this provides the possibility to change the ratio of the crystal acceleration to the acceleration of the neutrons. The results obtained at different frequencies were consistent within the experimental uncertainties. Thus it was not necessary to place the crystal in the same fluid as in the gravity experiment. A separation of the two effective mass peaks (0.9 mm) was observed at zero angular velocity, like in the gravity experiment. This value can be explained by a small constant bending of the crystal (0.053 μrad) which is in good agreement with a former result with the same crystal and identical geometry of the collimating slits.[5] The offset in the gravity experiment and the Coriolis experiment was not the same because the position of the collimating slits had to be changed. This has the result that a different part of the first crystal was used in both experiments. The angle between the lattice planes of the two crystals in these parts is slightly different compared to each other resulting in different offsets.

§.4 VCN interferometry

This Mach-Zehnder type interferometer consists of three quartz plates with sputter-etched transmission phase gratings with 2μm spacing.[9] The plates are mounted parallel to each other on an optical bench at present with a distance of 25 cm between successive gratings. For future experiments, the interferometer can be extended to a length of up to 6m.

In order to achieve sufficiently high fringe visibility, great care must be taken, in order to align the separately mounted gratings parallel to each other. This alignment is accomplished with three laser interferometers. The laser interferometers consist of three 100μm absorption gratings on the same quartz substrate as the neutron gratings.

The VCN interferometer setup avoids the major theoretical and experimental problems of the former COW experiments, namely bending of the silicon interferometer crystals under their own weight and dynamical diffraction effects in the crystals.

If the whole interferometer setup is tilted around the beam axis by an angle ϕ, the gravitational phase shift of the neutrons φ is given by

$$\varphi = \frac{2\pi m^2 g \lambda}{h^2} A \sin(\phi) \qquad (3)$$

Here m is the neutron mass, A the area enclosed by the interferometer paths and λ the de Broglie wavelength of the neutrons.

For the present VCN interferometer setup, a phase shift of 1 rad is induced by a tilt angle of only 0.05 degrees, compared to 1.15 degrees in the COW experiments. This increase in sensitivity is due to the higher wavelength at a comparable enclosed area. For an interferometer with an overall length of 4m, the tilt angle for the same phase shift will be in the order of a few arcseconds.

By applying a suitable electric DC field to the neutron beam inside the VCN interferometer, it is possible to detect an interaction of neutrons with an electric potential, which will result in an observable phase shift, if there is a non zero neutron charge. If one considers an interferometer with a length of 4 m and our recently measured values of intensity and fringe visibility, such an experiment could establish a new upper limit for the electric charge of the neutron in the order of $4 \cdot 10^{-22}$ electron charges.[6]

Bialynicki-Birula and Mycielski proposed an additional nonlinear term in the Schrödinger equation[7] which has the form $f(\psi) = -b \cdot \ln(|\psi|^2)$, where b is a new universal constant. Such a logarithmic term can be introduced without changing most of the well-established features of standard quantum mechanics. This nonlinearity can be measured by placing neutron absorbers in the interferometer paths at different places. As the absorber reduces $|\psi|^2$ the nonlinear term should produce a change in the k-vector and hence a measurable phase shift, if the absorber is moved along one neutron path. The upper limit for the magnitude of such a nonlinear term in the Schrödinger equation could be lowered from $3.3 \cdot 10^{-15}$eV (given by the experiments of Gähler et.al[8]) to about $1 \cdot 10^{-17}$eV. Since the sensitivity of this method depends on the flight time of the neutrons through the apparatus, the VCN interferometer is a sensitive device for this purpose.

Interference fringes with a visibility of 22% have been observed soon after the reinstallation of the VCN interferometer at the Institut Laue-Langevin (see Fig.4). The fringes were measured by moving one grating relatively to the others.

§.5 Conclusions

The deflection in the Coriolis experiment is the effect of an inertial force and in the gravity experiment of a gravitational force. As the results of both experiments depend on the numeric value of Planck's constant, this allows to compare the heavy and the inertial mass of the neutron in the quantum limit. The value of the ratio m_g/m_i

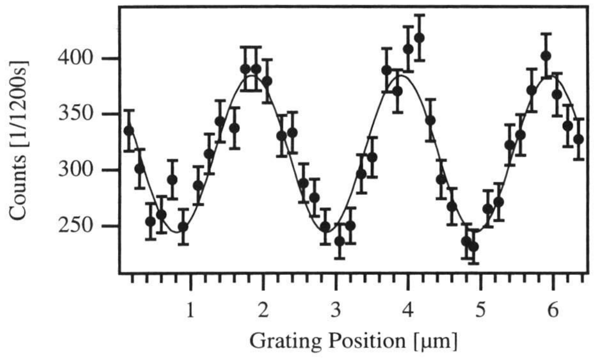

Fig. 4. Interference fringes measured with the VCN interferoemeter. The distance of the phase gratings was 25cm. The straight line is the result of a fit with the function $ff(x) = U + A$. $\sin(2\pi x/d + \varphi_0)$, with d being the spacing of the gratings of 2μm

obtained from the experimental results of both experiments is 1.011±0.015 . The accuracy is of the same order of magnitude which was achieved in the COW experiments, but is poor compared to experiments in the classical domain. The importance of these experiments for the validation of the equivalence principle in quantum mechanics provides a strong motivation for us to measure gravitational phase shifts with the VCN interferometer with higher accuracy. We should be able to reach a level of uncertainty in the 0.1% region. The experiment is currently in preparation.

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