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Aging Phenomena of Magnetization in the Multi-layer Random Energy Model

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From many experiments on aging with temperature variation, peculiar aging properties of spin glasses, i.e., *rejuvenation* and *memory* effect, are revealed recently. In this manuscript, we study the aging in the Multi-layer Random Energy Model by simulations and clarify that how these properties emerge in this model.

KEYWORDS: aging, multi-layer random energy model, hierarchical picture, memory effect, rejuvenation

§1. Introduction

It is well known that in disordered systems such as spin glasses, orientational glasses, polymer glasses and disordered ferromagnetics, relaxation processes to the equilibrium are very slow and the aging, dynamical behavior largely depending on history of system after quench from above the transition temperature T_c, is observed. Recently, by many experiments on aging phenomena *with temperature variation*,[1–7] very peculiar aging properties are revealed in spin glasses. Fig. 1 taken from ref. 4 is an example of these experiments. In this experiment, sample is quenched from above T_c to below and out-of-phase ac-susceptibility χ'' is observed during a negative temperature cycling procedure. From this figure, we can see that although χ'' seems to saturate its equilibrium value to some extent during t_1, χ'' relaxes very strongly during t_2 as if the system *rejuvenated* and forgot the aging at $T = 12$K. From this result, one may consider that the equilibrium of spin glasses changes chaoticly with temperature[8,9] and the system tries to approach the equilibrium at $T = 10$K which is very different from the one at $T = 12$K. But this explanation is clearly contradictory to the *memory* effect, i.e., χ'' resumes its relaxation from the value at the end of the period t_1 when the system comes back at $T = 12$K after the waiting time t_2 at $T = 10$K. Because χ'' seems to approach roughly to the equilibrium value at $T = 10$K during t_2, one can expect that the system reaches near the equilibrium at this temperature in this period. If the equilibrium at $T = 10$K is quite different from the one at $T = 12$K, strong relaxation toward the equilibrium at $T = 12$K should be observed during t_3 and memory effect would not appear.

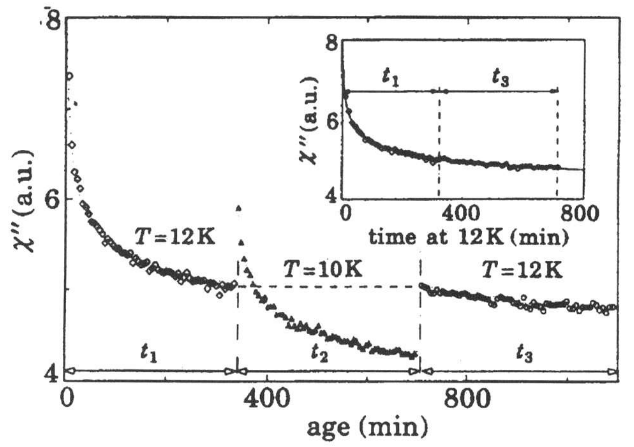

Fig. 1. Out-of-phase susceptibility χ'' to ac-field (f=0.01Hz) is observed during the negative temperature cycling in $CdCr_{1.7}In_{0.3}S_4$ spin glass ($T_g = 16.7$K). In inset, the t_1 and t_3 parts of data are connected after omitting the t_2 part. The solid line is a relaxation at $T = 12$K without temperature cycling.

In this manuscript, we introduce magnetization to the Multi-layer Random Energy Model (MREM)[10–12] ,which has a hierarchical structure causing continuous phase transitions, and perform some Monte Carlo simulations to observe relaxation of χ'' during the temperature cycling procedure. From the simulations, we confirmed that the MREM reproduces the results of the experiment well and the continuous transitions play important rules for the rejuvenation and the memory effect, as discussed in the hierarchical picture.

§2. Model

In this section, we explain the MREM briefly and introduce magnetization to this model. As shown in Fig. 2, this model is constituted by piling up L layers hierarchically. The bottom points represent the possible states of the system. The barrier energy $E(\alpha_n)$ for system to be activated from α_{n-1} to α_n (where α_n is the n-th ancestor of α) is given randomly and independently according to the distribution

$$\rho_n(E_n) = \exp[-E_n/T_c(n)]/T_c(n), \qquad (2.1)$$

where $T_c(n)$ is the transition temperature of the n-th layer counted from the bottom and satisfies inequalities $T_c(1) < T_c(2) < \cdots < T_c(L)$. Therefore the system undergoes continuous transitions as the system is cooled

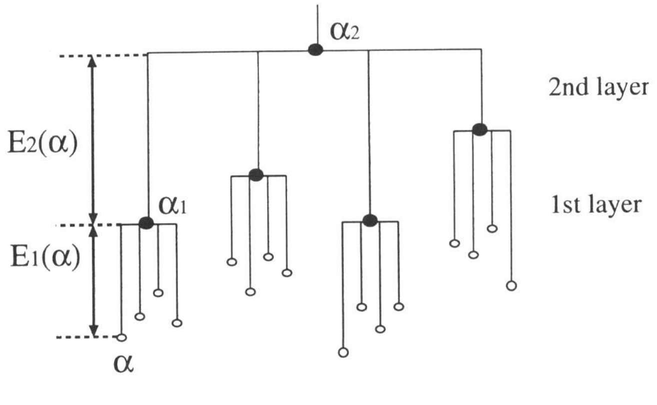

Fig. 2. Structure of the Multi-layer Random Energy Model with $L = 2$.

down. These continuous transitions correspond to the branching process of the free energy in the so-called hierarchical picture.[1,4,5] From the Arrhenius law, the average relaxation time of n-th layer $\langle\tau(n)\rangle$ is calculated by using this distribution as

$$\langle\tau(n)\rangle = \int_0^\infty \mathrm{d}E\rho_n(E)\tau_0\exp(E/T) = \begin{cases} \dfrac{\tau_0 T}{T - T_c(n)} & (T > T_c(n)), \\ \infty & (T \le T_c(n)), \end{cases} \quad (2.2)$$

where τ_0 is a microscopic time scale. This suggests that a transition from an ergodic phase to a non-ergodic phase occurs at $T_c(n)$ in the n-th layer. For dynamics, we employ the following simple Markoff process:[13]

(i) The system is activated from a state α to α_n with probability $W(\alpha, n)$ in unit time. This hopping probability is given as

$$W(\alpha, n) = \tau_0^{-1}\left\{\prod_{k=1}^{n}\exp[-E_k(\alpha)/T]\right\} \times (1 - \exp[-E_{n+1}(\alpha)/T]), \quad (2.3)$$

where $E_{L+1}(\alpha) \equiv \infty$. Note that the first factor on the right hand represents the probability that the system can be activated from α to α_n and the second one represents the system can not be activated from α_n to α_{n+1} respectively.

(ii) The system falls into a state chosen from all the states under α_n with equal probability and return to (i).

Next we introduce magnetization to the MREM to observe the magnetic response. It is natural to suppose that the nearer two states are located in the phase space the stronger the correlation of the two magnetizations is, and that the distance in the MREM can be measured in terms of barrier height. In fact the barrier height is related with the overlap in the SK model.[14] To incorporate this aspect, we assign the value of magnetization to state α, M_α, as

$$M_\alpha = \mathcal{M}_0(\alpha) + \mathcal{M}_1(\alpha_1) + \cdots + \mathcal{M}_{L-1}(\alpha_{L-1}), \quad (2.4)$$

where $\mathcal{M}_k(\alpha_k)$ is a contribution from the branch point α_k given as an independent random variable with zero mean. The correlation between M_α and M_β with the lowest common branch point α_k comes from the common contributions of $\mathcal{M}_n$ ($n = k, k+1, \cdots$) to these magnetizations. It decreases monotonically as k increases and the barrier becomes higher. In this simulation, we choose the same uniform distribution for $\mathcal{M}_n$.

The Zeeman energy due to applying magnetic field $H(t)$ is attributed to the lowest layer and $E_1(\alpha)$ in eq. (2.3) is replaced with $E_1(\alpha) + H(t)M_\alpha$.

§3. Simulations

In the following we show results of simulations performed on the MREM with $L = 2$, $T_c(1) = 0.6$ and $T_c(2) = 1.0$. See ref. 13 for the details of simulational

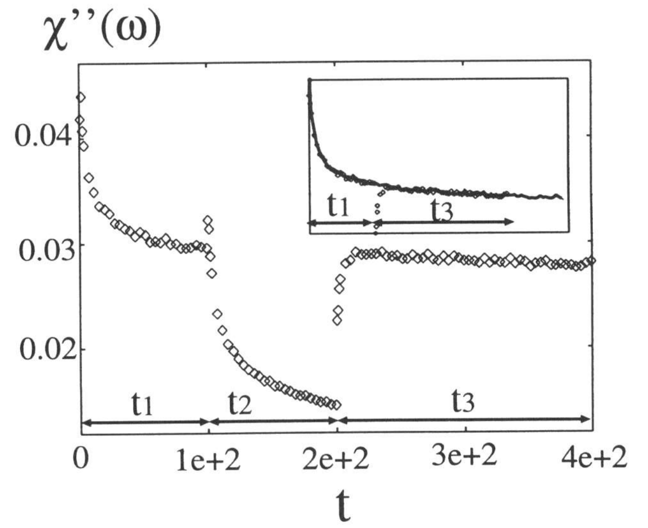

Fig. 3. χ'' observed during the negative temperature cycling. In the inset, the t_1 and t_3 parts of data are connected and conpared with unperturbed ($t_2 = 0$) data.

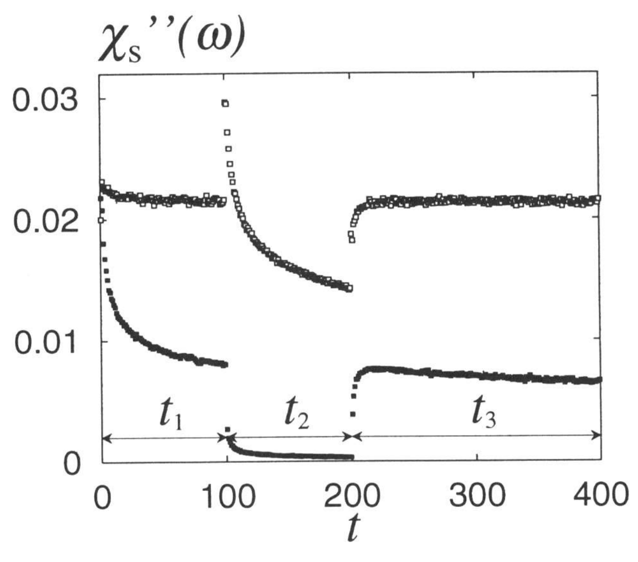

Fig. 4. The relaxation of χ_0'' (open squares) and χ_1'' (full squares).

procedure. The number of samples used for random average is typically 10^7. The amplitude and the period of the applying ac-field are fixed to 0.1 and $100\tau_0$, respectively. Hereafter this period is used as the unit of time. We prepare the initial condition so that the system is quenched from an infinitely high temperature. In other words, the initial state is chosen randomly.

In Fig. 3, we show a result of the negative temperature cycling simulation. In order that transitions occur with temperature variations in the first layer, the two temperatures $T = 0.85$ and 0.5 are set to satisfy

$$0.5 < T_c(1) < 0.85 < T_c(2).$$

From the inset, we can see the both relaxations at $T = 0.85$ are connected well except the beginning of the third stage (during t_3). χ'' seems to reach its equilibrium value enough in the first stage, and small jump and subsequent strong relaxation are observed in the second stage, as seen in experiments.

In Fig. 4, we plot χ_0'' and χ_1'' which are evaluated from $\mathcal{M}_0$ and $\mathcal{M}_1$, respectively ($\chi'' = \chi_0'' + \chi_1''$). We can see that χ_0'' remains almost constant in the first and third stages because $T > T_c(1)$ so that the first layer quickly relaxes to the equilibrium. The long-time dependence of relaxation is dominated by the second layer there. In contrast the first layer is dominant in the second stage because $T = 0.5$ is too low for the second layer to be activated and χ_1'' is much smaller than χ_0''. From these results, we notice that the dominant layer changes as the temperature varies.

We can also see that the slight jump of χ'' and the strong relaxation in the second stage are brought from χ_0''. This rejuvenation of χ_0'' is due to the quenching of the first layer across its transition temperature $T_c(1)$.

In order to investigate which states contribute to χ'' in more detail, we examine the energy distribution $P_n(E_n, t)$ which is defined as the probability density of finding the system at time t in one of the states whose energy of the n-th layer is E_n. By the definition, we have $P_n(E_n, t = 0) = \rho_n(E_n)$ since the initial state is chosen randomly.

In Fig. 5, we show time dependence of $P_n(E_n, t)$ in

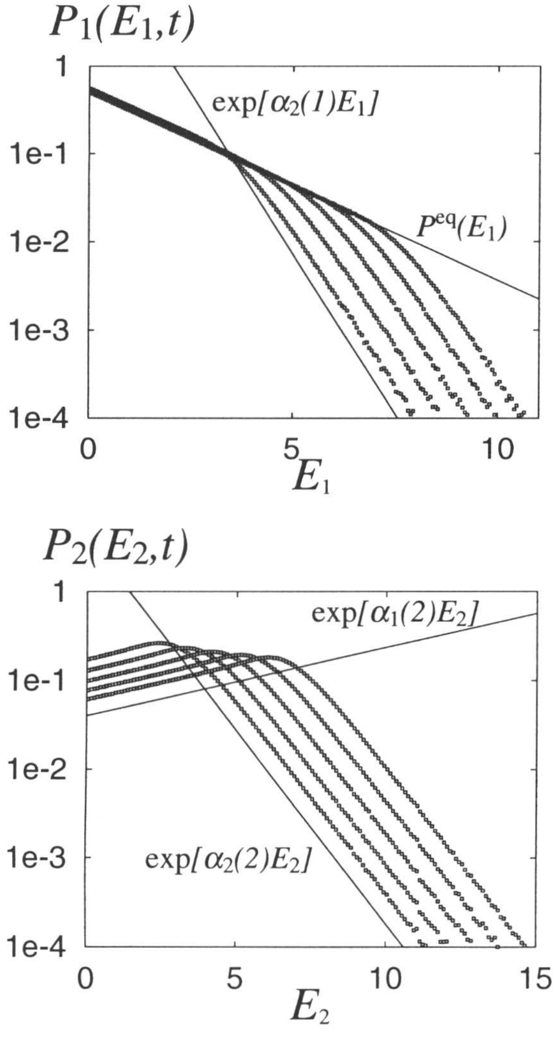

Fig. 5. $P_n(E_n, t)$ in the first stage at $t = 1, 10^{0.5}, 10, \ldots, 10^2$ (from left to right).

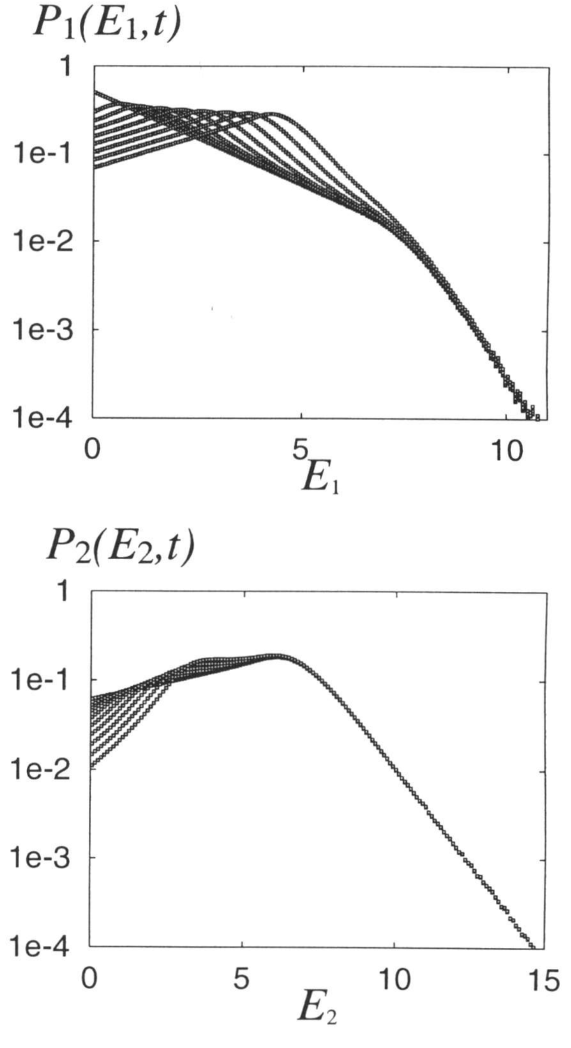

Fig. 6. $P_n(E_n, t)$ in the second stage at $t = 0, 10^{-1.5}, 10^{-1}, \ldots, 10^2$ (from left to right).

the first stage. We can see that the distribution consists of two exponential functions which are connected at a certain point, E^*. The value of E^* roughly represents the energy level up to which the system can be activated during the time interval t, and is estimated as

$$E^* \approx T \log[t/\tau_0]. \tag{3.1}$$

For $E_n \leq E^*$ the distribution is aged or equilibrated, so that the exponent $\alpha_1(n)$ is given as

$$\alpha_1(n) = \frac{1}{T} - \frac{1}{T_c(n)}, \tag{3.2}$$

while the other part $E_n \geq E^*$ leaves untouched and the exponent $\alpha_2(n)$ is equal to that of $\rho_n(E)$, i.e.,

$$\alpha_2(n) = -\frac{1}{T_c(n)}. \tag{3.3}$$

For the first layer we have $\alpha_1(1) < 0$ since $T > T_c(1)$, and $P_1(E_1, t)$ converges to the equilibrium distribution $P^{\rm eq}(E_1) = |\alpha_1(1)| \exp\{-|\alpha_1(1)|E_1\}$ very quickly (Fig. 5, upper). Note that the discrepancy between $P^{\rm eq}(E_1)$ and $P_1(E_1, t)$ for $E_1 > E^*$ is negligible in magnitude. For the second layer where $\alpha_1(2) > 0$, on the other hand, $P_2(E_2, t)$ has a peak at E^* moving to the right with time (Fig. 5, lower). This is why χ_1'' continues to change with age while χ_0'' remains almost constant in the first stage.

The experiments on the Zero-Field-Cooled (ZFC) magnetization in spin glasses[15,16] have shown that the distribution of the relaxation time τ, which has a peak at $\tau_{\rm max}$, depends on the waiting time $t_{\rm w}$ and $\tau_{\rm max}$ nearly coincides with $t_{\rm w}$, which just corresponds to the shift of the peak shown in the lower of Fig. 5.

In Fig. 6, we show time dependence of $P_n(E_n, t)$ in the second stage. We can check that a peak appears and moves to the right for the first layer (Fig. 6, upper). This new aging process is reflected as the strong relaxation of χ_0'' in the second stage as seen before. As for $P_2(E_2, t)$, the global aspects such as the position of peak formed in the first stage change little although the distribution of the lower energy decreases gradually. This brings the memory effect to this model.

Finally let us comment on the positive temperature cycling. We observed rejuvenation at the third stage when the system is heated up across $T_c(1)$ durint t_2. This is because that the peak of $P_1(E_1, t)$ created in the first stage is quickly destroyed at the second stage. The details are discussed in ref. 13.

§4. Conclusions and Discussions

We have shown that the MREM can reproduce the prominent behavior found in spin glasses, i.e., *rejuvenation* and *memory* effect when transitions occur at a certain layer, even if the number of layers is only two. Now let us discuss what happens when $L \gg 1$ and summarize the mechanism to appear these property in the MREM. For a given temperature $T < T_c(L) = T_c$, there exists the n-th layer such that $T_c(n-1) < T < T_c(n)$. The essential point is the fact that layers below n are quickly equilibrated and do not contribute to the long time relaxation, while those above n are almost quenched and they behave as if the time evolution stops. In this sense the n-th layer is an *activated* one and the long time relaxation at temperature T is dominated by this layer. As the system ages at T, the peak position of energy distribution of this activated layer shifts to right and this position is stored as the information of aging at this temperature. This *memory* is reserved while the system is cooled and is destroyed by heating up above $T_c(n)$. If L is large enough, a small temperature variation brings a shift of the activated layer. Therefore we can observe a new aging process in a new activated layer by cooling the system a little from T. This is the origin of rejuvenation in this model. During the system stays at this lower temperature, the activated layer at T is almost *frozen* and the memory stored in this layer is reserved. Therefore when the system is heated back to T and starts the dynamical process according to the memory stored in the activated layer, the system remembers the aging in the first stage.

It is challenging to find some hierarchical structure, if really exists in spin glasses, to which the real spin space is mapped and to investigate that these prominent aging properties of spin glasses emerge along this scenario.

1) Ph. Refreggier, E. Vincent, J. Hammann and M. Ocio: J. Phys. (Paris) **48**(1987)1533.
2) P. Granberg, L. Sandlund, P. Nordblad, P. Svedlindh and L. Lundgren: Phys. Rev. B **38**(1988)7097.
3) P. Granberg, L. Lundgren and P. Nordblad: J. Magn. Magn. Mater **92**(1990)228.
4) F. Lefloch, J. Hammann, M. Ocio and E. Vincent: Europhys. Lett **18**(1992)647.
5) E. Vincent, J. P. Bouchaud, J. Hammann and F. Lefloch: Phil. Mag. B **71**(1995)489.
6) J. O. Andersson, J. Mattsson and P. Nordblad: Phys. Rev. B **48**(1993)13977.
7) K. Jonason, E. Vincent, J. Hammann, J. P. Bouchaud and P. Nordblad: Phys. Rev. Lett. **81**(1998)3243.
8) A. J. Bray and M. A. Moore: Phys. Rev. Lett. **58**(1987)57.
9) D. S. Fisher and D. A. Huse: Phys. Rev. Lett. **56**(1986)1601.
10) J. P. Bouchaud and D. S. Dean: J. Phys. I France **5**(1995)265.
11) B. Derrida: J. Phys. Lett. France **46**(1985)401.
12) B. Derrida and E. Gardner: J. Phys. C **19**(1986)2253.
13) M. Sasaki and K. Nemoto: preprint cond-mat/0002024.
14) K. Nemoto: J. Phys. A **21**(1988)L287.
15) P. Svedlindh, P. Granberg, P. Nordblad, L. Lundgren and H. S. Chen: Phys. Rev B **35**(1987)268.
16) L. Lundgren, P. Svedlindh and O. Beckman: Phys. Rev B **26**(1982)3990.